

APPENDIX B

CALCULATING AVERAGE IQ DECREMENT ASSUMING A NON-ZERO THRESHOLD ON THE IQ/BLOOD-LEAD CONCENTRATION RELATIONSHIP

This appendix is an update to Appendix E1 of the §403 risk analysis report, which provided details on how the health effect and blood-lead concentration endpoints are calculated given that blood-lead concentration is lognormally distributed with a geometric mean and geometric standard deviation specified by GM and GSD, respectively. In estimating average IQ decrement due to lead exposure and the percentages of children whose IQ decrement as a result of lead exposure was at or above 1, 2, or 3 points, the §403 risk analysis (as detailed in Appendix E1) assumed an average IQ decrement of 0.257 points for every 1.0 µg/dL increase in blood-lead concentration, and that no blood-lead threshold existed in this relationship (i.e., no non-zero blood-lead concentration existed below which the predicted IQ decrement was zero). To evaluate how the assumption of no threshold affects the estimates of these IQ decrement parameters, the sensitivity analyses presented within Chapters 5 and 6 of this document includes analyses that estimate these parameters under specified assumptions on a non-zero threshold (Sections 5.1.4 and 6.2.2). This appendix shows how these estimates were calculated in these sensitivity analyses (i.e., given a non-zero threshold). (Note that the assumption of a threshold does not affect how the probability of having a blood-lead concentration at or above a specified value or the probability of observing an IQ less than 70 due to lead exposure are calculated.)

P[IQ decrement $\geq x$] for $x=1, 2, 3$

Let Y denote the IQ decrement associated with a blood-lead concentration specified by PbB . Assume that the non-zero blood-lead threshold in the blood-lead/IQ relationship is denoted by T . Then

$$\begin{aligned}
 Y &= 0.257*(PbB - T) && \text{when } PbB \geq T \\
 &= 0 && \text{when } PbB < T.
 \end{aligned}$$

Thus, for any positive value x , the probability of observing an IQ decrement (Y) at or above x is determined by the following:

$$P[Y \geq x] = P[0.257*(PbB-T) \geq x] = P[PbB \geq (x/0.257 + T)] = P[\ln(PbB) \geq \ln(x/0.257 + T)]$$

where $\ln(\cdot)$ denotes the natural logarithm transformation. Then, since PbB is assumed to have a lognormal distribution,

$$P[\text{IQ decrement} \geq x] = 1 - \Phi \left(\frac{\ln\left(\frac{x}{0.257} + T\right) - \ln(\text{GM})}{\ln(\text{GSD})} \right)$$

where $\Phi(z)$ is the probability of observing a value less than z under the standard normal distribution.

Average IQ decrement

Under the same notation as in the previous paragraph, let $f(x)$ denote the probability density function (PDF) of PbB (i.e., the PDF of a lognormal distribution), let $F(x)$ denote the cumulative density function (CDF) of PbB (i.e., $F(x) = P[\text{PbB} \leq x]$), and let $g(y)$ denote the PDF of Y . Then

$$\begin{aligned} g(y) &= (1/0.257) \cdot f(y/0.257 + T) && \text{when } y > 0 \\ &= F(T) && \text{when } y = 0 \end{aligned}$$

Then, the average IQ decrement, denoted by $E[Y]$, is given by

$$E[Y] = \int_0^{\infty} y \cdot f(y/0.257 + T) \cdot (1/0.257) dy = [0.257 \int_T^{\infty} x \cdot f(x) dx] - [0.257 \cdot T \int_T^{\infty} f(x) dx]$$

This equates to the following:

$$\begin{aligned} \text{Avg. IQ decrement} = E[Y] = & \\ & 0.257 \cdot \text{GM} \cdot \exp\left(\frac{\ln(\text{GSD})^2}{2}\right) \cdot \left[1 - \Phi\left(\frac{\ln(T) - \ln(\text{GM}) - \ln(\text{GSD})^2}{\ln(\text{GSD})}\right)\right] \\ & - 0.257 \cdot \left[1 - \Phi\left(\frac{\ln(T) - \ln(\text{GM})}{\ln(\text{GSD})}\right)\right] \end{aligned}$$

Note that when $T=0$, average IQ decrement = $0.257 \cdot \text{GM} \cdot \exp(\ln(\text{GSD})^2/2)$, which is equation (4) specified within Appendix E1 of the §403 risk analysis report.

The standard deviation of the distribution of IQ decrement (Y) equals

$$\text{S.D. (IQ decrement)} = \sqrt{E(Y^2) - [E(Y)]^2}$$

The value of E[Y] is given above, and the value of E(Y²) can be found to equal

$$\begin{aligned} E[Y^2] = & 0.257^2 \cdot \left\{ \exp(2(\ln(\text{GM}) + \ln(\text{GSD})^2)) \cdot \left[1 - \Phi \left(\frac{\ln(\text{T}) - \ln(\text{GM})}{\ln(\text{GSD})} - 2\ln(\text{GSD}) \right) \right] \right. \\ & - 2\text{T} \cdot \exp(\ln(\text{GM}) + \ln(\text{GSD})^2 / 2) \left[1 - \Phi \left(\frac{\ln(\text{T}) - \ln(\text{GSD})}{\ln(\text{GSD})} - \ln(\text{GSD}) \right) \right] \\ & \left. + \text{T}^2 \cdot \left[1 - \Phi \left(\frac{\ln(\text{T}) - \ln(\text{GM})}{\ln(\text{GSD})} \right) \right] \right\} \end{aligned}$$