

Populations, Activity and Emissions of Diesel Nonroad Equipment in EPA Region 7

BSFC Calculation Methodology Appendix AD

Assessment and Standards Division
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To determine brake-specific emissions at a point in time (actually a one-second interval) and engine speed (or RPM), N , during the test, we first determine the engine output power, $Power_{actual}(N)$, at that point in time. The engine output power is the maximum power, $Power_{max}(N)$, times the percent load demand, $Load(N)$, that is,

$$Power_{actual}(N) = Power_{max}(N) * Load(N) \quad [eqn 1]$$

Because most of the measurements did not include engine control unit data the engine speed, N , and percent load, $Load(N)$, or actual engine power, $Power_{actual}(N)$, needed to be estimated. In most cases the data measured were limited to gaseous emissions and exhaust flow rates. From these however, the actual fuel rate, $FuelRate_{actual}(N)$, could be estimated from a carbon balance¹⁻⁴

$$FuelRate_{actual}(N) = (\{ [CO_2]_{exhaust} - [CO_2]_{ambient} \} * MW_{CO_2} + [CO] * MW_{CO} + [HC_1] * MW_{HC}) * m_{exhaust} / MW_{exhaust} \quad [eqn 2]$$

where $[CO_2]_{exhaust}$, $[CO_2]_{ambient}$, $[CO]$, and $[HC_1]$ are the measured exhaust CO_2 and ambient CO_2 , CO , and hydrocarbon concentrations, MW_{CO_2} , MW_{CO} , MW_{HC} , and $MW_{exhaust}$ are the molecular weights of CO_2 , CO , H_1C , and the exhaust gas, respectively, and $m_{exhaust}$ is the reported exhaust mass flow. The vehicle fuel rate can also be expressed explicitly in terms of the power demand on the engine⁵⁻¹⁴,

$$FuelRate = \frac{1}{LHV} \cdot \left(\frac{k \cdot N \cdot V_d}{2000} + \frac{P}{\eta_i} \right) \quad [eqn 3]$$

which includes the vehicle tractive power, P , engine friction, $kN V_d/2000$, indicated efficiency, η_i , and the lower heating value of the fuel, LHV . In this relationship k is the average piston speed, S_p , and engine speed, N , dependent engine friction^{15,16},

$$k = k_0 + k_1 N + k_2 \overline{S_p}^2 \quad [eqn 4]$$

At maximum power the fuel rate is expected to be at its maximum, and the ratio of the fuel rate at maximum power to the maximum power, P_{max} , is referred to as the brake specific fuel consumption (bsfc)

$$bsfc(N) = FuelRate_{max}(N) / P_{max}(N). \quad [eqn 5]$$

Using the above relationships for fuel rate and power,

$$\begin{aligned}
\frac{FuelRate}{FuelRate_{max}} &= \frac{\frac{1}{LHV} \cdot \left(\frac{k \cdot N \cdot V_d}{2000} + P \right) / \eta_i}{\frac{1}{LHV} \cdot \left(\frac{k \cdot N \cdot V_d}{2000} + P_{max} \right) / \eta_i} = \frac{\left(\frac{k \cdot N \cdot V_d}{2000} + P \right) / \eta_i}{\left(\frac{k \cdot N \cdot V_d}{2000} + P_{max} \right) / \eta_i} = \frac{\left(\frac{\eta_i k \cdot N \cdot V_d}{2000} + P \right)}{\left(\frac{\eta_i k \cdot N \cdot V_d}{2000} + P_{max} \right)} \\
&= \frac{\frac{\eta_i k \cdot N \cdot V_d}{2000}}{\left(\frac{\eta_i k_{max} \cdot N \cdot V_d}{2000} + P_{max} \right)} + \frac{P}{\left(\frac{\eta_i k_{max} \cdot N \cdot V_d}{2000} + P_{max} \right)} \\
&= \frac{\frac{\eta_i k \cdot N \cdot V_d}{2000}}{P_{max} \left(1 + \frac{\eta_i k_{max} \cdot N \cdot V_d}{2000 P_{max}} \right)} + \frac{P}{P_{max} \left(1 + \frac{\eta_i k_{max} \cdot N \cdot V_d}{2000 P_{max}} \right)} \\
&= \frac{\eta_i k \cdot N \cdot V_d}{2000 P_{max}} \left(1 - \frac{\eta_i k_{max} \cdot N \cdot V_d}{2000 P_{max}} + \left(\frac{\eta_i k_{max} \cdot N \cdot V_d}{2000 P_{max}} \right)^2 - \left(\frac{\eta_i k_{max} \cdot N \cdot V_d}{2000 P_{max}} \right)^3 + \dots \right) + \\
&\quad \frac{P}{P_{max}} \left(1 - \frac{\eta_i k_{max} \cdot N \cdot V_d}{2000 P_{max}} + \left(\frac{\eta_i k_{max} \cdot N \cdot V_d}{2000 P_{max}} \right)^2 - \left(\frac{\eta_i k_{max} \cdot N \cdot V_d}{2000 P_{max}} \right)^3 + \dots \right) \\
&\approx \frac{P}{P_{max}} \left(1 - \frac{\eta_i k_{max} \cdot N \cdot V_d}{2000 P_{max}} \right) + \frac{\eta_i k \cdot N \cdot V_d}{2000 P_{max}} \\
&= \frac{P}{P_{max}} + \frac{\eta_i k \cdot N \cdot V_d}{2000 P_{max}} \left(k - \frac{P}{P_{max}} k_{max} \right)
\end{aligned} \tag{eqn 6}$$

The frictional terms for a diesel engine are on the order of 5 to 15% of the total engine load depending on the engine speed and load¹⁴⁻¹⁶. In this approximation the engine friction contribution, i.e.,

$$\left(k - \frac{P}{P_{max}} k_{max} \right) \times \left(\frac{\eta_i k_{max} \cdot N \cdot V_d}{2000} \right) / P_{max} \tag{eqn 7}$$

will be less than 15% and will have the greatest values near the minimum load points. Hence, to calculate brake specific emissions the approximation

$$FuelRate(N) / FuelRate_{max}(N) \approx P(N) / P_{max}(N) \tag{eqn 8}$$

or

$$\begin{aligned}
P(N) &\approx FuelRate_{actual}(N) * [P_{max}(N) / FuelRate_{max}(N)] \\
&= FuelRate_{actual}(N) / BSFC_{max}(N)
\end{aligned} \tag{eqn 9}$$

is used to estimate the vehicle engine power where $FuelRate_{actual}(N)$ is the actual amount of fuel used at engine speed N , both obtained from SEMTECH output.

The Semtech $FuelRate(N)$ output is in units of gal/sec which is converted to grams/second. The conversion uses the following constants: Density H_2O @ 70 F = 62.3 lbm/ft³, 1 ft³ = 7.481 gal, 1 lbm = 0.4536 kg, SG=0.85. So the overall conversion constant is $62.3 * 0.4536 * 1000 * 0.85 / 7.481 = 3210.85$ g/gal. Because the BSFC units are typically in

g/kW-hr and the fuel rate is in units of g/s there is an additional conversion factor needed. That conversion factor is 3600 s/hour.

$$Power_{actual}(N) = FuelRate_{actual}(N) [g/s] * 3600 / BSFC_{max}(N) [g/kW-hr] \quad [eqn 10]$$

To calculate the brake-specific mass emissions over a specified period of time the work done by the engine must be determined. At each sample interval of $\Delta t_i = 1$ second, the work done by the engine is

$$Work_{actual,i} = Power_{actual,i}(N) \times \Delta t_i \quad [eqn 11]$$

From this the total work over the test can be calculated by summing over all test times,

$$Work_{test total} = \left(\sum_{all\ test\ sample\ times,\ i} Work_{actual,i}(N) \right) \quad [eqn 12]$$

Substituting in equation 11, yields the total work done during a test in terms of the engine power at each test interval,

$$Work_{test total} = \left(\sum_{all\ test\ sample\ times,\ i} Power_{actual,i}(N) \times \Delta t_i \right) \quad [eqn 13]$$

The units of choice for work, is [kW-hr] and as mentioned above the sample times are all, $\Delta t_i = 1$ second. So to get the work in terms of [kW-hr] a factor of 1hour / 3600 seconds must be included. That is,

$$Work_{test total} [kW - hr] = \left(\sum_{all\ test\ sample\ times,\ i} Power_{actual,i}(N) \times \Delta t_i \right) \times \frac{1[second]}{3600[seconds / hour]} \quad [eqn 14]$$

Similarly, the cumulative gaseous emissions in unit of grams over the test are calculated as:

$$Emissions_{test total} = \left(\sum_{all\ test\ sample\ times,\ i} Emissions_{actual,i}(N) \times \Delta t_i \right) \quad [eqn 15]$$

where each $Emissions_{actual,i}(N)$ is a 1 hz emission value at engine speed, N, and time interval, Δt_i , obtained from the SEMTECH output. The SEMTECH software also calculates 1hz time resolved brake specific emissions in units of g/kW-hr. The work-equivalent (brake-specific) emissions over a given test are thus

$$Emissions_{brake\ specific} [grams/kW-hr] = Emissions_{test total} / Work_{test total} \quad [eqn 16]$$

For particulate matter (PM) mass emissions, the value for the test total mass, $m_{test total}^{filter}$, is collected in aggregate, i.e., on a filter, but must be corrected for partial sampling and dilution over the specified period of time. The filter

mass, $m_{test\ total}^{filter}$, is the post-test minus the pre-test filter mass difference which is measured in the laboratory on a gravimetric scale¹⁷. From the test measurements, the vehicle exhaust test total PM mass is

$$m_{test\ total}^{exhaust} = \rho_{test\ total}^{exhaust} \times V_{test\ total}^{exhaust} \quad [\text{eqn 17}]$$

where $\rho_{test\ total}^{exhaust}$ is the cumulative exhaust mass concentration or mass density and $V_{test\ total}^{exhaust}$ is the vehicle total test exhaust volume. The test total exhaust PM mass density can be put in terms of the test total filter mass, $m_{test\ total}^{filter}$, and the test total MPS sample volume, $V_{test\ total}^{MPS\ sample}$, as follows

$$\begin{aligned} \rho_{test\ total}^{exhaust} &= \rho_{test\ total}^{MPS\ sample} && \text{since the sample is collected directly from the exhaust} \\ &= m_{test\ total}^{MPS\ sample} / V_{test\ total}^{MPS\ sample} && \\ &= m_{test\ total}^{filter} / V_{test\ total}^{MPS\ sample} && \text{since the filter collects all of the MPS sample volume} \\ &&& \text{and all dilution air is HEPA filtered} \end{aligned} \quad [\text{eqn 18}]$$

The test total sample volume, $V_{test\ total}^{MPS\ sample}$, is calculated by summing over all test time the measured MPS sample flow rates, $(\Delta V / \Delta t)_i^{MPS\ sample}$, which are in units of standard $\text{cm}^3/\text{minute}$ and then applying appropriate conversion factors to get the total test MPS sample volume in units of liters,

$$V_{test\ total}^{MPS\ sample} [L] = \left(\sum_{\text{all test sample times, } i} (\Delta V / \Delta t)_i^{MPS\ sample} [scm^3 / minute] \times \Delta t_i \frac{1000 [L / cm^3]}{60 [seconds / minute]} \right) \quad [\text{eqn 19}]$$

Similarly, the test total exhaust volume, $V_{test\ total}^{exhaust}$, can be calculated by summing over the individual measured exhaust flow rates, $(\Delta V / \Delta t)_i^{exhaust}$,

$$V_{test\ total}^{exhaust} [m^3] = \left(\sum_{\text{all test sample times, } i} (\Delta V / \Delta t)_i^{exhaust} [scft^3 / minute] \times \Delta t_i \times 0.0283 [m^3 / ft^3] \right) \quad [\text{eqn 20}]$$

Hence, the vehicle exhaust test total PM mass is

$$\begin{aligned} m_{test\ total}^{exhaust} &= \rho_{test\ total}^{exhaust} \times V_{test\ total}^{exhaust} \\ &= \left(m_{test\ total}^{filter} / V_{test\ total}^{MPS\ sample} \right) \times V_{test\ total}^{exhaust} \end{aligned} \quad [\text{eqn 21}]$$

And the work-equivalent (brake-specific) emissions are calculated as:

$$m_{test\ total}^{exhaust} / Work_{test\ total} [kW\ hr] = \frac{m_{test\ total}^{filter} \times (V_{test\ total}^{exhaust} / V_{test\ total}^{MPS})}{Work_{test\ total} [kW\ hr]} \quad [eqn\ 22]$$

where the test total work, $Work_{test\ total}$, is given in equation 14.

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