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To: WIPP Records Center

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Subject: Discussion of Predicted Long-Term Heat Conduction due to Proposed SDI Tests at WIPP

This memo discusses and extends the results of the AP-156 analysis report (Kuhlman, 2011), with the aim of addressing EPA comments on the Planned Change Notice (PCN) submitted by DOE regarding the long-term effects of the Salt Disposal Investigation (SDI) thermal tests proposed at WIPP (DOE, 2011). Some generalities related to heat conduction in solids are restated, to clarify the proposed long-term thermal effects presented in (Kuhlman, 2011). For comparison with the proposed SDI thermal tests, some historic tests from WIPP are summarized.

1.1 Relevant Summary of Thermal Analysis Report for AP-156

An analytical heat conduction solution was used to conservatively estimate the rise in temperature at the WIPP waste disposal panels due to the proposed SDI heater tests (Kuhlman, 2011). The calculation used a well-known two-dimensional analytical solution and the method of superposition. These solutions and methods are found in standard heat conduction textbooks.

The analytical solution assumes heat conduction is two-dimensional, forcing the heat pulse to diffuse horizontally away from the heaters; no heat conducts above or below the elevation of the testing area. Like most sedimentary rocks, the halite at WIPP is not homogeneous, it consists of layers that have large horizontal extent but are relatively thin. Both clay and anhydrite layers exist in the Salado Formation, which have lower thermal conductivity than pure halite. The two-dimensional flow assumption confines heat conduction to a horizontal plane, even though in reality some heat will flow up and down through anhydrite and clay layers. The two-dimensional flow assumption is conservative because it increases the predicted temperature at the waste panels.

The analytical solution does not take into account the excavations that exist in the halite, because doing so would greatly complicate the relatively simple scoping solution. The air in excavations will have very low thermal conductivity and heat capacity, compared to intact halite (radiation will be the important heat transfer mechanism in air). To accurately consider the thermal evolution near the heaters, it would be critical to take the three-dimensional nature of heat conduction around the drifts into account. The effect of the excavations is not considered important to the conclusions of the current analysis.

Heat conduction in halite is known to behave in a slightly non-linear manner, due to the temperature-dependence of thermal conductivity; as temperature rises, thermal conductivity decreases. The change in thermal conductivity is actually slight, since the exponent in the proposed functional behavior is approximately -1, specifically $k(T) = 5.4(300/T)^{1.14}$ (see Kuhlman, 2011 or Stone et al., 2010). This relationship indicates that a doubling of temperature will approximately cut thermal conductivity in half. Thermal conductivity often varies over orders of magnitude; except very near the heat source it can essentially be considered to be a constant. The non-linear behavior will slow down heat conduction away from the heaters in the short term, but will not have much effect on long-term heat conduction predictions.

The AP-156 analysis report (Kuhlman, 2011) documents the calculation, material properties, and temporal and geometrical arrangement used. In this report, Section 5 lists the slightly modified Python script used to compute and plot the figure shown in this report, allowing the calculations to be checked and verified.

1.2 Limiting Behavior of Thermal System

The limiting behavior of the system can be studied by examining the limiting behavior of the analytical solution. At large distance, the solution will appear similar to that from a single point source of equivalent strength. The small-scale geometry of the area immediately surrounding the sources is unimportant at late time and large distance. At late times, the temporal configuration of the heat source is equivalent to a short constant-strength heat source. The short-time variability in heat sources is unimportant at late time and large distance.

Slight modifications were made to the Python script presented in the AP-156 analysis report, to also plot a time-variable set of sources (Figure 1) and a single effective source (replacing the five sources used in the AP-156 analysis report) for comparison.

Three scenarios are plotted in Figures 1 and 3. In Figure 1, the different line types correspond to the three scenarios explained in Figure 2, while the line colors correspond to different radial distances away from the center of the SDI experiment area (for $x \geq 0$ along the line $y = 0$). For observation locations at least 100 m from the SDI heaters or observation times at least 20 years after the tests began, the results of the three scenarios are basically the same.

In Figure 3, the three line types correspond to the same three scenarios as in Figure 1, while the line colors correspond to different observation times. As in Figure 1, radial distances correspond to a line heading south (along $y = 0$) towards the disposal panels. The curves also show that for any location after at least 20 years, the three scenarios are basically the same.

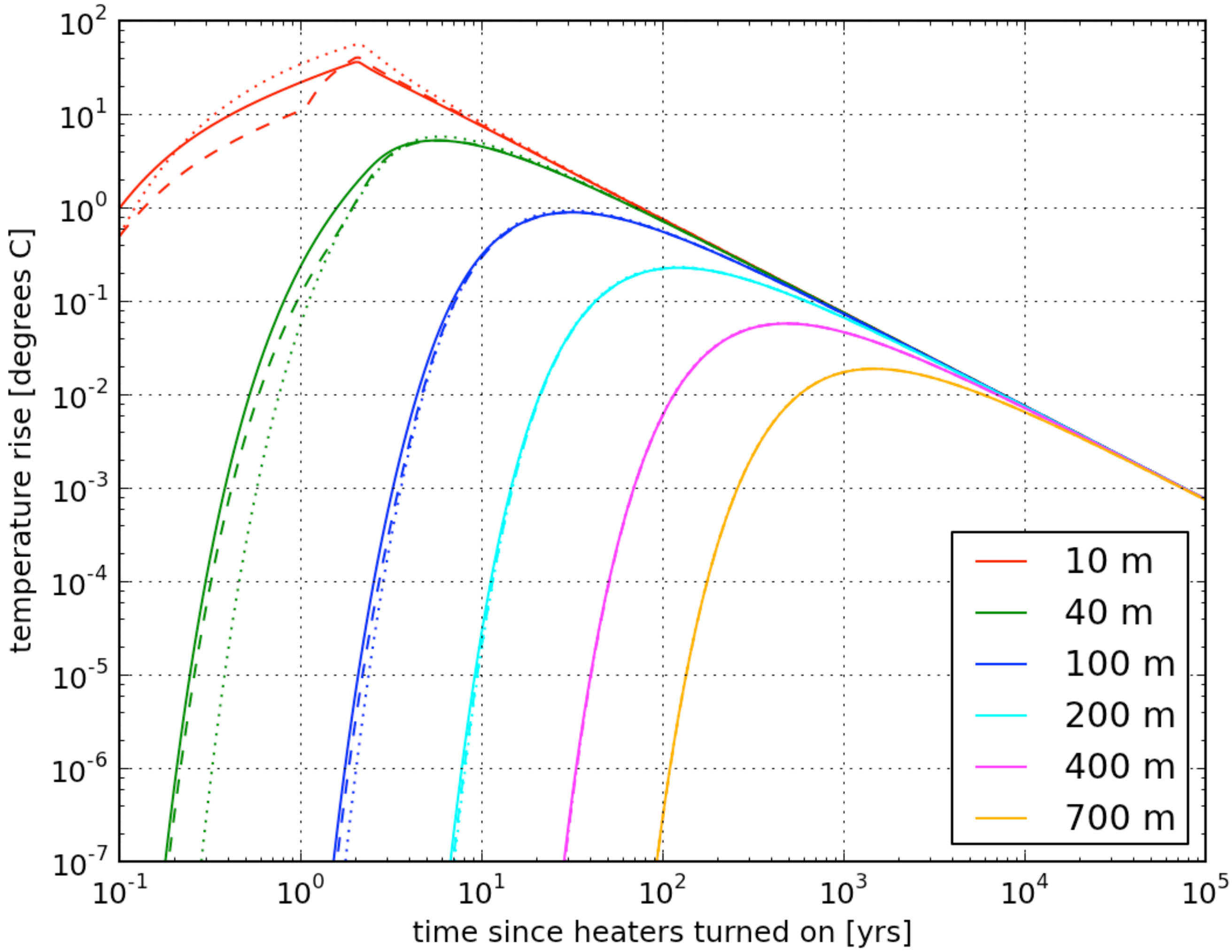
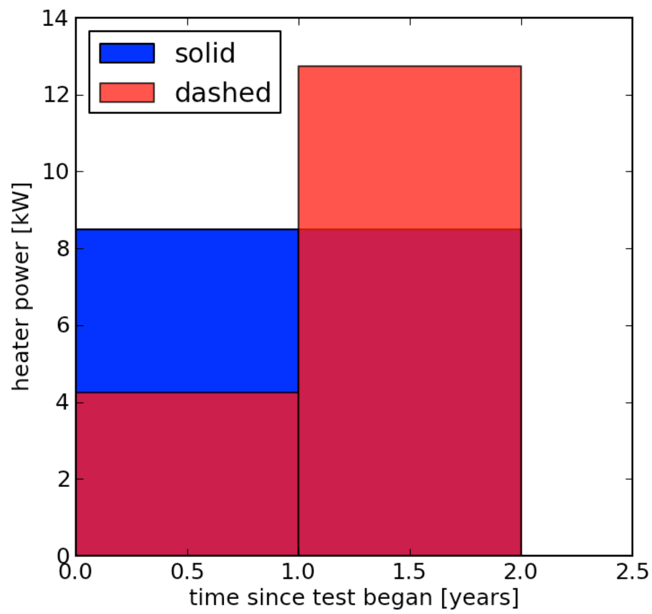


Figure 1. *Predicted temperature rise due to proposed SDI thermal test at six radial distances and three scenarios. Solid lines represent five heaters with constant output over two years, same as (Kuhlman, 2011). Dashed lines represent five heaters with variable output over two years. Dotted lines represent one heater with constant output over two years. See Figure 2 for explanation of different source scenarios.*



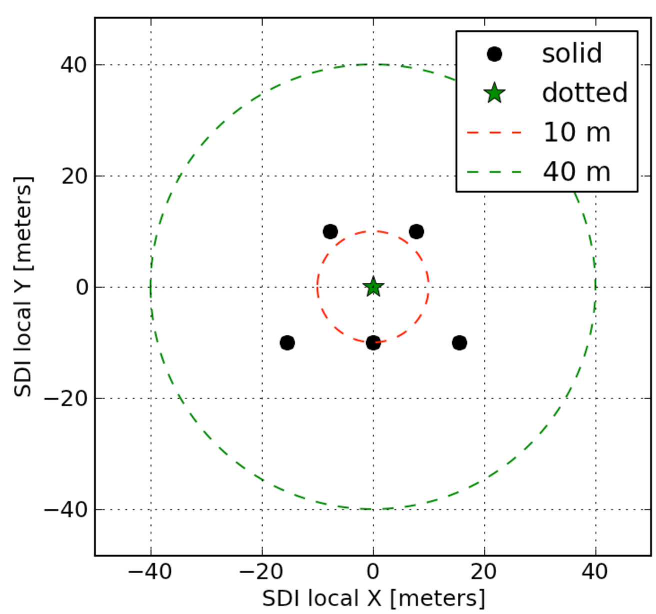


Figure 2. *Illustrations of three different source-configuration scenarios in Figures 1 and 3. Left bar chart shows temporal variability in source between solid and dashed lines in Figures 1 and 3. Right map shows spatial locations of five proposed sources and single effective source in Figures 1 and 3. Dashed circles correspond to the two shortest distance curves plotted in Figure 1.*

Figure 1 illustrates that near-field effects and short-term variability are unimportant when considering long-term heat conduction. The energy balance done in the AP-156 analysis report computed the predicted long-term (i.e., steady state) rise in temperature for a finite domain, given the heat capacity of the halite and the total energy added.

The total energy proposed to be added in the SDI experiments is that due to five 8.5 kW heaters operating for two years and is given by

$$E = (5\text{ heaters}) \times \left(8.5\ \frac{\text{kW}}{\text{heater}}\right) \times (2\text{ years}) = 85\text{ kW-years.}$$

This energy is spread over a cylinder of halite with a 700 meter radius and a 16.67 meter height. The expected volume-averaged rise in temperature can be compute from

$$E = \rho V C \Delta T$$

where ρ is the density of salt [2190 kg/m^3], C is heat capacity of salt [931 J/(kg·K)], V is the volume of salt [$\pi \times (700\text{ m})^2 \times 16.67\text{ m} = 2.566\times10^7\text{ m}^3$], and ΔT is the resulting average temperature rise [5.13E-2 degrees Celsius]. The material properties of WIPP halite are taken from Stone et al., (2010). This calculation does not include the thermal conductivity, because conductivity only affects the rate at which heat is conducted, not the long-term predicted temperature.

Figure 3 shows thermal rise profiles in space at different times for the same three different source configurations discussed in Figures 1 and 2. The profile at 2000 years shows an almost constant temperature rise of 0.025 degrees C at all distances. This prediction is slightly lower than that found with the energy balance above because the analytical solution considers an infinite domain; some of the heat has flowed out beyond the circle with 700 m radius assumed above in the energy balance calculation. The steady-state temperature rise for an infinite domain, due to a finite pulse of energy is zero.

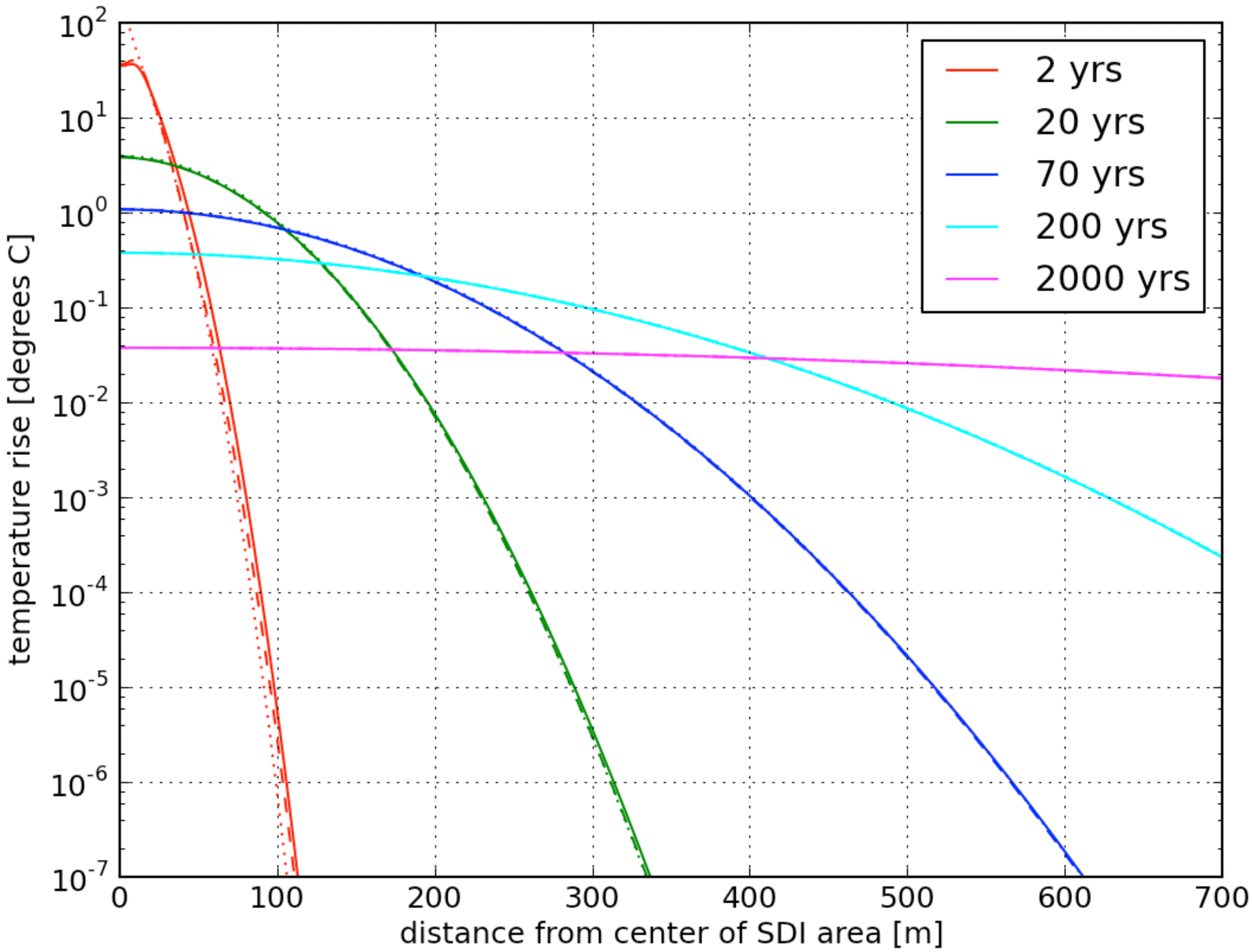


Figure 3. *Predicted temperature rise due to proposed SDI thermal test at five times and three scenarios. Solid lines represent case with five heaters with constant strength for two years, same as (Kuhlman, 2011). Dashed lines represent five heaters with variable strength. Dotted lines represent one heater with constant strength for two years. See Figure 2 for explanation of different source scenarios.*

1.3 Historic In-Situ Heater Tests at WIPP

Previous studies regarding the potential long-term thermal effects of a full-scale high-level waste repository have been summarized (Tyler et al., 1988; §2.2.3). The studies predicted "little sensible temperature rise at any of the locations of concern [several hundred meters away], even for the highest thermal loads" (p. 52). The amount of heat that is proposed in the SDI tests is orders of magnitude less than the high-level waste disposal scenarios considered and over a very short time period, compared to permanent disposal. Any effects of the heater tests themselves would be even more insignificant.

There have been several in situ heater tests at WIPP in the past (Figure 4), including Defense High-Level Waste (DHLW) studies done in Rooms A and B, the Materials Interface Interaction Test (MIIT) in Room J, Contact- and Remote-Handled (CH & RH) TRU waste tests, and a heated pillar experiment in Room H. Both Matalucci (1990) and Tyler et al. (1988) §2 summarize the suite of in situ Thermal/Structural Interaction (TSI) tests done at WIPP, including tests done in Rooms A, B and H. These references also summarize the in situ Waste Package Performance Tests (WPP) done at WIPP, including the MIIT tests (Room J), the CH/RH TRU waste container tests (Rooms T and J), and some aspects of tests in Rooms A and B. The CH/RH TRU WPP tests were relatively low-power tests (300 W or less) and are not discussed further.

The locations of the A Rooms relative to other experimental rooms were chosen to minimize thermal and mechanical effects that might propagate to other experiments in the underground (Munson et al., 1992; p. 7):

> *Rooms A1 and A3 are separated from other experimental rooms by approximately 79.8-m (262-ft) thick barrier pillars [of intact halite] which assures thermal and mechanical isolation from the other experimental rooms of the WIPP facility for the planned five year duration of the test.*

A similar statement was made about the design location of Room B (Munson et al., 1990; p. 6).

The tests in Room A included 68 heaters spanning three rooms, A2 the center room and A1 and A3 being guard rooms on either side. The cylindrical heaters were placed in vertical holes in the floor of the rooms, and were backfilled with crushed salt. Room A2 had 0.47 kW heaters, with triple-strength heaters (1.41 kW) at the ends of the room and in the neighboring guard rooms. The total power of the 18 W/m^2 Room-A experiments was on average 63.9 kW (Munson et al., 1992; p. 486). The heaters were operational in the Room A experiments for more than 4.5 years. The heat input in just the A Rooms is 150% of the proposed SDI thermal load for more than twice as long a period than that proposed in the SDI thermal tests.

The tests in Room B included 17 1.8-kW TSI heaters, eight 1.5-kW WPP heaters, four 4.0-kW TSI guard heaters, and four heat-generating glass sources, all placed in vertical boreholes. The total power of the Room B Overtest experiment was 58.6 kW (Munson et al., 1990; p. 751), or 137% of the proposed SDI test thermal load. The entire set of heaters in Room B were operational for more than 2.5 years, at this point the WPP canisters were recovered. The remaining heaters continued to run for at least another two years at about 110% of the proposed SDI rate. The Room B experiment was designed to consider higher-than-design heat load, different heater construction materials (some with intentional material defects), and different backfill materials, such as mixtures of crushed salt and vermiculite (Matalucci, 1990).

The Room H heated pillar experiment used strip heaters and insulation to keep the surface of a cylindrical 11-m diameter pillar at approximately 75° C for ten years. There were nine 1.42 kW heaters and one 1.2 kW heater, for a combined power of nearly 14 kW for the Room H experiment (Munson et al., 1987; p. 315), which is approximately one-third the proposed thermal load of the SDI tests.

The MIIT WPP test consisted of many different waste types suspended in 50 brine-filled 8-cm boreholes in the floor of Room J that were maintained at 90° C for up to five years (Tyler et al., 1988; p. 225).

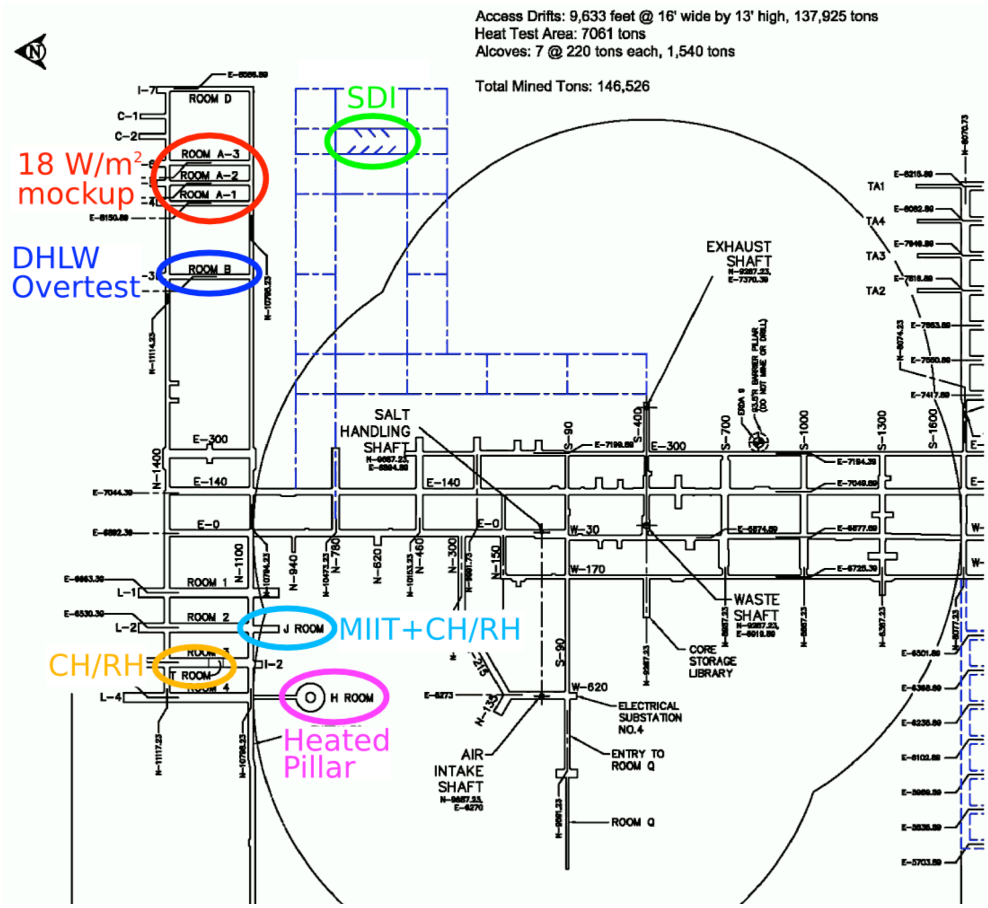


Figure 4. *Locations of in situ heated experiments at WIPP; background image from (DOE, 2011; Fig. 3-10).*

In summary, there have been several in situ tests at WIPP already that have historically imparted much more energy to the halite than proposed in the SDI thermal tests. Considering only the DHLW tests in Rooms A and B, approximately 288% the total SDI thermal load was added over a period of more than 2.5 years, then 260% the total SDI thermal load was added for another two years after that. To a lesser degree, the WPP tests in Rooms T and J, and the heated pillar experiment in Room H all added additional thermal load to the WIPP salt for several years.

No long-term large-scale heat buildup was predicted to occur before theses tests were conducted, nor has any been observed to occur since the tests were completed. The SDI test proposes to use five high-power (8.5 kW) heaters for a short period of time (DOE, 2011) and cannot realistically have more of an impact than the larger historical in situ tests.

2 Questions Regarding AP-156 Report

The EPA posed four questions regarding the long-term thermal analysis presented in the AP-156 thermal analysis report. These questions are grouped and their key points are addressed.

2.1 SDI HC-1 and HC-2

These two questions deal with the small-scale variability of the thermal properties of the heater/crushed-salt system. The questions indicated that the thermal conductivity of crushed salt is less than that of intact halite. The questions also mentioned that heat conduction in halite is non-linear because thermal conductivity is inversely proportional to temperature (SDI HC-2). The issue is centered on the assumption that these small-scale variations in thermal conductivity will slow down heat conduction and somehow lead to a higher temperature at the waste disposal panels.

The discussion in Section 1.2 (including Figures 1 and 3) of this report illustrates that small-scale variability in the exact location and temporal nature of the heat source have very little effect on any predictions at late time or large distance.

The non-linearity of heat conduction in halite was already addressed in the AP-156 thermal report (Kuhlman, 2011). Except for very close to the heaters, the change in thermal properties will be small and the non-linearity will be slight. The analytical solution was not intended to accurately predict temperatures rise in the vicinity of the heaters. The solution is two-dimensional and does not take the variable properties associated with crushed salt or excavations into account. Ignoring this small-scale variability does not impact the prediction of temperature rise at large distance or late time (see Figures 1 and 3 and the related discussion).

The proposal that heat may build up within an insulating layer around the heaters, and therefore reach a higher temperature a long distance away is physically impossible. First, the crushed salt will only be on top of the heaters, they will be laid down on essentially intact salt. In heat conduction, energy takes the path of least resistance; if there is an insulating blanket of crushed salt on top of the heaters, more of the heat energy will conduct downward into the floor. Secondly, even if the heaters were completely surrounded by material of lower thermal conductivity, these differences thermal conductivity may change the rate at which heat conducts away from the heaters, but in no way does it effect the ultimate temperature that the system will attain (see energy balance argument in §1.2 of this report). As an analogy, imagine placing hot coffee in both a paper cup and an insulated thermos. For each case the ultimate long-term temperature that the coffee and the surrounding room attain are the same, the coffee in the thermos just takes longer to cool down than the coffee in the paper cup.

2.2 SDI HRP-1 and HRP-2

These two questions deal with the expected effects that a 300° C heat pulse would have on the properties of the clay seam and anhydrite marker beds in the Salado at WIPP.

The surface of the heater itself may rise to high temperatures, but any large increases in temperature should be confined to a volume of salt very close to the heaters. The analytical solution given in the report associated with AP-156 (Kuhlman, 2011) is not an appropriate solution to evaluate these factors. The results of a coupled thermal/mechanical finite element modeling analysis for the generic salt repository study are given in the SDI proposal (DOE, 2011); the figure is repeated here in Figure 5.

Figure 5 shows qualitatively that at the end of the two-year heater test, the highest temperatures will be confined to an area close to the heaters (nearly circular red and yellow areas that are <10 meters across). These calculations take the geometry of the sources and excavations into account and show that the heat pulse will be asymmetric (teardrop-shaped green and teal areas extending more to the left than to the right) due to the lower thermal conductivity of the crushed salt and air-filled excavation.

In response to the two questions, the large-magnitude thermal pulse will be confined to a very small area, and likely will not extend to effect Marker Beds 138 and 139 above and below the repository. On a very local scale, the heat may affect clay seams that intersect the excavations and are very near the heaters. These changes will be on a scale too small to incorporate any potential effects in a meaningful way into PA codes. At the scale that would be representable in PA codes, the changes to the material properties of the halite and its clay or anhydrite interbeds would be insignificant. Additionally, these material effects due to large temperature changes at the small scale are unknown, but will be studied during the proposed SDI tests.

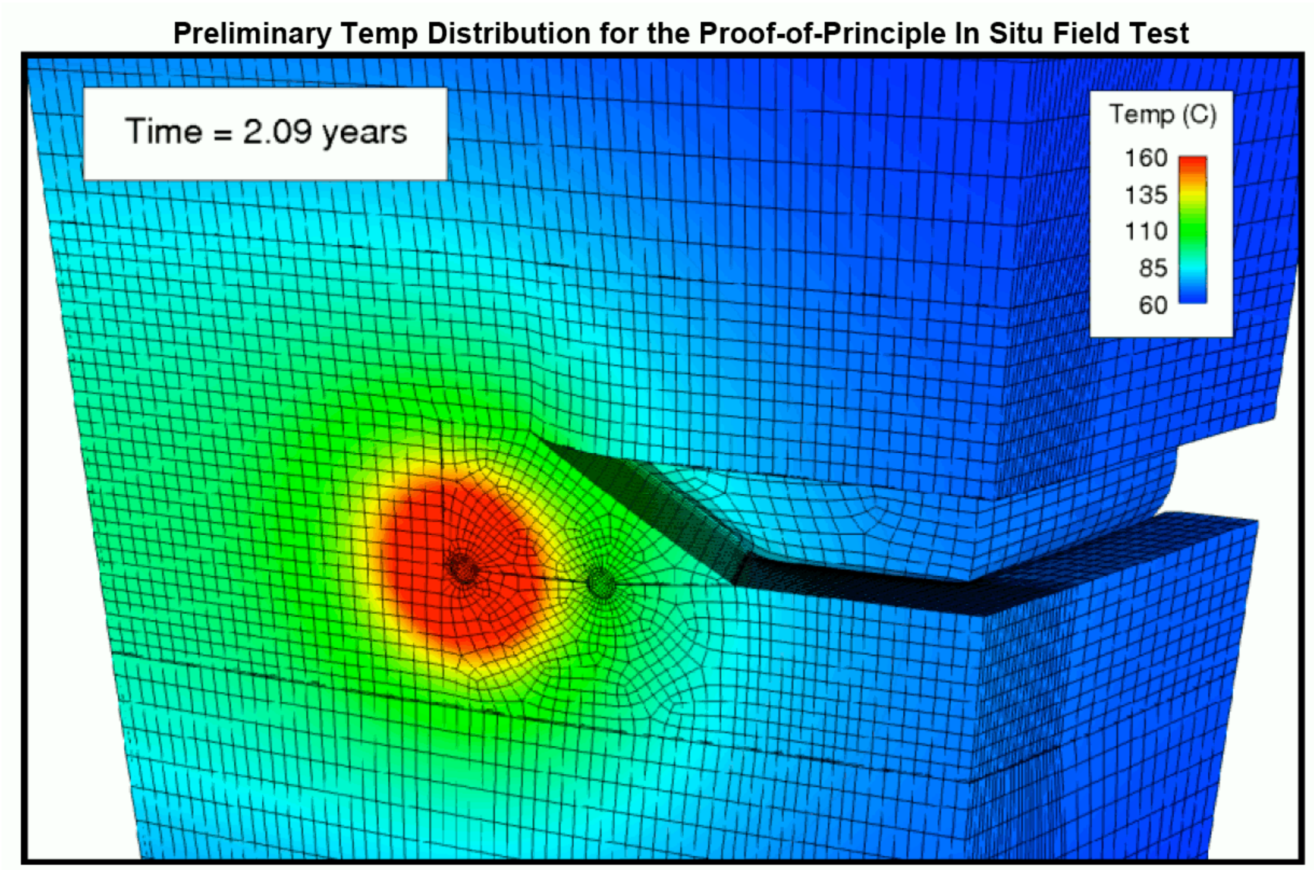


Figure 5. *Finite element thermal/mechanics analysis results (DOE, 2011; Figure 3-2); for scale, circles representing canister cross-sections are 2 ft in diameter. The contours represent a simulation that accounts for both the different thermal properties of the intact halite and crushed salt and the temperature-dependence of the halite thermal conductivity.*

3 Conclusions

The effects of two years of five 8.5 kW heaters in the SDI thermal tests will be insignificant at the location of the waste disposal panels (Panel 1 being the closest at approximately 700 m away) for any time. The two-dimensional analytical solution presented in the AP-156 analysis report (Kuhlman, 2011) and the modified analyses presented here do not take into account the crushed salt that will be placed over the heaters, or the excavations that will exist. These limitations of the solution used here do not invalidate its use as a screening tool. These analyses are bounding because they assume all the energy imparted to the salt through the heaters will only dissipate through conduction, which is a very slow transport mechanism. In reality, the drifts will be ventilated at the end of the tests to allow personnel to enter them and retrieve equipment and inspect and document the effects of the test. Once the drifts are ventilated with cool air, the large-scale thermal gradient will reverse and the hot wall rock will now conduct heat to the cooler drifts, rather than to the waste panels, 700 m away.

4 References

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5 Python Script Listing

The following Python script is a modified version of that used in the AP-156 analysis report (Kuhlman, 2011); it was used in the report to compute the solution and plot figures.

```python
# this script is part of the SNL SDI proposal scoping work
# by Kristopher L. Kuhlman, Repository Performance Dept. (6212)

# Oct-2011
# this script has been slightly modified (two routines added H2 and heaters2)
# which are very slight variations on the routines used before;
# the contour map was removed and an explanation figure was added.

import numpy as np               # array functionality
from scipy.special import exp1   # exponential integral
import matplotlib
matplotlib.use('Agg')
import matplotlib.pyplot as plt  # plotting functionality

def G(a1,f1,t1,r1):
    """2D solution for line source

    a1 = thermal diffusivity [W/(m*K)]
    t1 = 1D time vector   [s]
    r1 = radial distance (any shape >= 1D) [m]
    """

    oldshape = list(r1.shape)
    nt = t1.shape[0]
    r1.shape = (1,-1)  # reform into 1D vector with singleton second dim
    t1.shape = (-1,1)  # make t conformable with r

    Z1 = f1/(4.0*np.pi*a1)*exp1(r1**2/(4.0*a1*t1))

    # change inputs back to original shape
    r1.shape = oldshape
    t1.shape = (nt,)

    # reshape result so it has dimensions like r
    # with the t dimension added in front
    oldshape.insert(0,nt)

    Z1.shape = oldshape
    return Z1

def H(a2,f2,t2,tau2,r2):
    """use superposition in time to compute a
    source that is non-zero boundary flux from 0 <= t <= tau

    a2,k2,t2,r2 are same as in G()
    tau2 = time heaters turn off [s]
    f2 = actual flux strength [W]

    NB: routine assumes times are listed in increasing order
    """

    # source on at t=0
    T0 = G(a2,f2,t2,r2)

    tt = t2-tau2 # shifted times

    # number of non-zero times at beginning of vector
    nnz = (tt[:] < 0).sum()

    # sink on at t=tau (only positive times are valid)
    T1 = G(a2,f2,tt[nnz:],r2)

    # combine source and sink
```

```
    T2 = np.empty_like(T0)
    T2[:nnz] = T0[:nnz]          # before heater turns off
    T2[nnz:] = T0[nnz:] - T1[:]  # after heater turns off
    return T2

def H2(a2,f2,t2,tau2,r2):
    """use slightly more complicated superposition in time to compute
    source that is half-strength from 0 <= t <= tau/2 and
    3/2 strength from tau/2 <= t <= tau and zero for t > tau

    added October, 2011

    a2,k2,t2,r2 are same as in G()
    tau2 = time heaters turn off [s]
    f2 = actual flux strength [W]

    NB: routine assumes times are listed in increasing order
    """

    # half-strength source on at t=0
    T0 = G(a2,f2/2.0,t2,r2)

    tt = t2-tau2/2.0 # tau/2 shifted times
    ttt = t2-tau2  # tau shifted times

    # number of non-zero times at beginning of vector
    nnz1 = (tt[:] < 0).sum()
    nnz2 = (ttt[:] < 0).sum()

    # regular strength source on at t=tau/2 (1.5 total with T0)
    T1 = G(a2,f2,tt[nnz1:],r2)
    # 3/2-strength sink on at t=tau (only positive times are valid)
    T2 = G(a2,f2*1.5,ttt[nnz2:],r2)

    # combine source and sink
    T3 = np.empty_like(T0)
    T3[:] = T0[:]   # half-strength heater (0.5 strength for tau/2 years)
    T3[nnz1:] += T1[:]  # add in full-strength heater (1.5 strength total for tau/2 years)
    T3[nnz2:] -= T2[:]  # after heater turns off (zero strength from tau to infinity)
    return T3

def heaters(a3,f3,t3,tau3,xg,yg,htrs):
    """ use superposition to in horizontal (x,y) to sum up
    effects of multiple heaters installed at different x,y locations,
    assuming all heaters are at the same elevation.

    a4,k4,t4 are same as G()
    tau4,f4 are same as H()
    xg,yg are arrays of observation coordinates [m]
    source terms are located at complex coordinates passed
    in the list htrs (heaters) [m].
    """

    Wshape = list(xg.shape)
    Wshape.insert(0,t3.shape[0])
    W3 = np.zeros(Wshape,dtype=np.float64)

    Zg = xg + yg*1j

    for i,heat in enumerate(htrs):

        # compute relative horizontal (2D) distance from heater
        rg = np.abs(Zg - heat)
        W3 += H(a3,f3,t3,tau3,rg)

    return W3

def heaters2(a3,f3,t3,tau3,xg,yg,htrs):
    """ use superposition to in horizontal (x,y) to sum up
    effects of multiple heaters installed at different x,y locations,
```

```
    assuming all heaters are at the same elevation.

    added October, 2011
    same as heaters(), but calls H2() instead of H()

    a4,k4,t4 are same as G()
    tau4,f4 are same as H()
    xg,yg are arrays of observation coordinates [m]
    source terms are located at complex coordinates passed
    in the list htrs (heaters) [m].
    """

    Wshape = list(xg.shape)
    Wshape.insert(0,t3.shape[0])
    W3 = np.zeros(Wshape,dtype=np.float64)

    Zg = xg + yg*1j

    for i,heat in enumerate(htrs):

        # compute relative horizontal (2D) distance from heater
        rg = np.abs(Zg - heat)
        W3 += H2(a3,f3,t3,tau3,rg)

    return W3


def heaters3(a3,f3,t3,tau3,xg,yg):
    """function added October, 2011
    similar to heaters() but replaces 5 heaters with a single central
    heater that is 5x as powerful

    a4,k4,t4 are same as G()
    tau4,f4 are same as H()
    xg,yg are arrays of observation coordinates [m]
    """

    Wshape = list(xg.shape)
    Wshape.insert(0,t3.shape[0])
    W3 = np.zeros(Wshape,dtype=np.float64)

    # compute relative horizontal (2D) distance from
    # center of SDI experiment (effective single heater)
    rg = np.abs(xg + yg*1j)
    W3 = H(a3,f3*5.0,t3,tau3,rg)

    return W3


# @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
# setup material properties

k = 5.4        # thermal conductivity  [Watt/(meter*Kelvin)]
alfa = 2.648E-6  # thermal diffusivity   [meter^2/second]
density = 2190.0  # density of salt [kg/m^3]
Cp = 931.0  # heat capacity of salt [Joule/(kilogram*Kelvin)]
strength = 8500.0   # power of each heater [Watt]
f0 = strength/(Cp*density*16.67)  # line source strength

# @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
# setup calculation grid and input parameters

secperyr = 365.2422*24.0*60.0*60.0  # seconds in a year

# time after t=0 heaters get turned off [2 years in seconds]
tau =   2.0 *secperyr  # end of heaters
maxt = 20.0 *secperyr  # "final" map calculation date (2035, assuming begins in 2015)

# Computational grid is with respect to SDI
# proposal figure (north is to left), so computational
```

```python
# x+ is South (x- is North), y+ is East (y- is West)

# compute out to 750m since it is about 680 m
# from proposed heater locations (as per SDI proposal) to panel 1
nt,nx,ny  = (21, 100, 100)
minx,miny = (-100.0, -750.0)
maxx,maxy = (750.0,  100.0)

tg = np.linspace(1.0E-6,maxt,nt)  # time [seconds]

# compute on a grid, with center of heater array at origin.
xg,yg = np.meshgrid(np.linspace(miny,maxy,ny),np.linspace(minx,maxx,nx))

# 3D mesh for plotting
X,Y = np.mgrid[minx:maxx:nx*1j, miny:maxy:ny*1j]

# @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
# perform calculation

# distances related to proposed geometry of heaters
# estimated from figures in SDI proposal.
hdew = 15.5     # east-west distance between heaters
hdns = 20.0/2.0 # half north-south distance between heaters

htrs = [hdns - hdew/2.0*1j,
        hdns + hdew/2.0*1j,
        -hdns -hdew*1j,
        -hdns + 0j,
        -hdns + hdew*1j]

if __name__ == "__main__":

    # @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
    # plot figures of results

    # compute solution for radial profile at different times
    xg = np.linspace(0,700,700)
    yg = np.zeros_like(xg)
    mint = 0.1
    maxt = 100000.0
    tg = np.array([2,20,70,200,2000])*secperyr

    T  =  heaters(alfa,f0,tg,tau,xg,yg,htrs)
    T2 = heaters2(alfa,f0,tg,tau,xg,yg,htrs)
    T3 = heaters3(alfa,f0,tg,tau,xg,yg)

    col = ['red','green','blue','cyan','magenta','orange','gray','purple','pink']

    fig = plt.figure(1)
    ax = fig.add_subplot(111)
    for i,tval in enumerate(tg):
        ax.semilogy(xg,T[i,:],'-',color=col[i],label='%.0f yrs' % (tval/secperyr,))
        ax.semilogy(xg,T2[i,:],'--',color=col[i])
        ax.semilogy(xg,T3[i,:],':',color=col[i])
    ax.set_ylim([1.0E-7,100])
    ax.set_xlabel('distance from center of SDI area [m]')
    ax.set_ylabel('temperature rise [degrees C]')
    plt.grid()
    plt.legend(loc='upper right')
    plt.savefig('temp-profile.png',dpi=150)
    plt.close(1)


    # compute solution at log-spacing of time
    xg = np.array([10.0,40.0,100.0,200.0,400.0,700.0])
    yg = np.zeros_like(xg)
    mint = 0.1
    maxt = 100000.0
    tg = np.logspace(np.log10(mint*secperyr),np.log10(maxt*secperyr),300)
```

```python
T  =  heaters(alfa,f0,tg,tau,xg,yg,htrs)
T2 = heaters2(alfa,f0,tg,tau,xg,yg,htrs)
T3 = heaters3(alfa,f0,tg,tau,xg,yg)

# plot temperature through time 50, 100, 200, 400, and 700 m east of heaters
fig = plt.figure(1)
ax = fig.add_subplot(111)
for i,xval in enumerate(xg):
    print i,xval
    ax.loglog(tg/secperyr,T[:,i],'-',color=col[i],label='%.0f m' % xval)
    ax.loglog(tg/secperyr,T2[:,i],'--',color=col[i])
    ax.loglog(tg/secperyr,T3[:,i],':',color=col[i])
ax.set_xlabel('time since heaters turned on [yrs]')
ax.set_ylabel('temperature rise [degrees C]')
ax.set_ylim([1.0E-7,100.0])
ax.set_xlim([mint,maxt])
plt.grid()
plt.legend(loc='lower right')
plt.savefig('temperature-through-time.png',dpi=150)
plt.close(1)

# plot small figures for illustrating temporal and spatial behavior of sources
fig = plt.figure(1,figsize=(12.0,5.0))

# left figure shows time behavior
ax1 = fig.add_subplot(121)

ax1.bar([0.0,1.0,2.0],[8.5,8.5,0],width=1.0,color='blue',label='solid')
ax1.bar([0.0,1.0,2.0],[8.5/2,8.5*3/2,0],width=1.0,color='red',
        alpha=0.75,label='dashed')
ax1.set_xlim([0,2.5])
plt.legend(loc='upper left')

ax1.set_xlabel('time since test began [years]')
ax1.set_ylabel('heater power [kW]')

# right figure shows effective single source
ax2 = fig.add_subplot(122)

for i,h in enumerate(htrs):
    if i == 0:
        ax2.plot(h.imag,h.real,'ko',markersize=8,label='solid')
    else:
        ax2.plot(h.imag,h.real,'ko',markersize=8)

ax2.plot(0.0,0.0,'g*',label='dotted',markersize=12)
theta = np.linspace(-np.pi,np.pi,200)
r = 10.0
ax2.plot(r*np.cos(theta),r*np.sin(theta),'r--',label='10 m')
r = 40.0
ax2.plot(r*np.cos(theta),r*np.sin(theta),'g--',label='40 m')
plt.grid()
plt.legend(numpoints=1)
ax2.axis('equal')
ax2.set_xlim([-50,50])
ax2.set_ylim([-50,50])
ax2.set_xlabel('SDI local X [meters]')
ax2.set_ylabel('SDI local Y [meters]')

plt.subplots_adjust(wspace=0.25)
plt.savefig('source-comparisons.png',dpi=150)
```