APPENDIX I

STATISTICAL TABLES AND PROCEDURES

I.1 Normal Distribution

Table I.1 Cumulative Normal Distribution Function Φ(z)

z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5674	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.6315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Negative values of z can be obtained from the relationship $\Phi(-z) = 1 - \Phi(z)$.

I.2 Sample Sizes for Statistical Tests

Table I.2a Sample Sizes for Sign Test

(Number of measurements to be performed in each survey unit)

(α,β) **or** (β,α)

	0.01	0.01	0.01	0.01	0.01	0.025	0.025	0.025	0.025	0.05	0.05	0.05	0.1	0.1	0.25
Δ/σ	**0.01**	**0.025**	**0.05**	**0.1**	**0.25**	**0.025**	**0.05**	**0.1**	**0.25**	**0.05**	**0.1**	**0.25**	**0.1**	**0.25**	**0.25**
0.1	4095	3476	2984	2463	1704	2907	2459	1989	1313	2048	1620	1018	1244	725	345
0.2	1035	879	754	623	431	735	622	503	333	518	410	258	315	184	88
0.3	468	398	341	282	195	333	281	227	150	234	185	117	143	83	40
0.4	270	230	197	162	113	192	162	131	87	136	107	68	82	48	23
0.5	178	152	130	107	75	126	107	87	58	89	71	45	54	33	16
0.6	129	110	94	77	54	92	77	63	42	65	52	33	40	23	11
0.7	99	83	72	59	41	70	59	48	33	50	40	26	30	18	9
0.8	80	68	58	48	34	57	48	39	26	40	32	21	24	15	8
0.9	66	57	48	40	28	47	40	33	22	34	27	17	21	12	6
1.0	57	48	41	34	24	40	34	28	18	29	23	15	18	11	5
1.1	50	42	36	30	21	35	30	24	17	26	21	14	16	10	5
1.2	45	38	33	27	20	32	27	22	15	23	18	12	15	9	5
1.3	41	35	30	26	17	29	24	21	14	21	17	11	14	8	4
1.4	38	33	28	23	16	27	23	18	12	20	16	10	12	8	4
1.5	35	30	27	22	15	26	22	17	12	18	15	10	11	8	4
1.6	34	29	24	21	15	24	21	17	11	17	14	9	11	6	4
1.7	33	28	24	20	14	23	20	16	11	17	14	9	10	6	4
1.8	32	27	23	20	14	22	20	16	11	16	12	9	10	6	4
1.9	30	26	22	18	14	22	18	15	10	16	12	9	10	6	4
2.0	29	26	22	18	12	21	18	15	10	15	12	8	10	6	3
2.5	28	23	21	17	12	20	17	14	10	15	11	8	9	5	3
3.0	27	23	20	17	12	20	17	14	9	14	11	8	9	5	3

Table I.2b Sample Sizes for Wilcoxon Rank Sum Test

(Number of measurements to be performed in the reference area and in each survey unit)

	(α,β) **or** (β,α)														
	0.01	**0.01**	**0.01**	**0.01**	**0.01**	**0.025**	**0.025**	**0.025**	**0.025**	**0.05**	**0.05**	**0.05**	**0.1**	**0.1**	**0.25**
Δ/σ	**0.01**	**0.025**	**0.05**	**0.1**	**0.25**	**0.025**	**0.05**	**0.1**	**0.25**	**0.05**	**0.1**	**0.25**	**0.1**	**0.25**	**0.25**
0.1	5452	4627	3972	3278	2268	3870	3273	2646	1748	2726	2157	1355	1655	964	459
0.2	1370	1163	998	824	570	973	823	665	440	685	542	341	416	243	116
0.3	614	521	448	370	256	436	369	298	197	307	243	153	187	109	52
0.4	350	297	255	211	146	248	210	170	112	175	139	87	106	62	30
0.5	227	193	166	137	95	162	137	111	73	114	90	57	69	41	20
0.6	161	137	117	97	67	114	97	78	52	81	64	40	49	29	14
0.7	121	103	88	73	51	86	73	59	39	61	48	30	37	22	11
0.8	95	81	69	57	40	68	57	46	31	48	38	24	29	17	8
0.9	77	66	56	47	32	55	46	38	25	39	31	20	24	14	7
1.0	64	55	47	39	27	46	39	32	21	32	26	16	20	12	6
1.1	55	47	40	33	23	39	33	27	18	28	22	14	17	10	5
1.2	48	41	35	29	20	34	29	24	16	24	19	12	15	9	4
1.3	43	36	31	26	18	30	26	21	14	22	17	11	13	8	4
1.4	38	32	28	23	16	27	23	19	13	19	15	10	12	7	4
1.5	35	30	25	21	15	25	21	17	11	18	14	9	11	7	3
1.6	32	27	23	19	14	23	19	16	11	16	13	8	10	6	3
1.7	30	25	22	18	13	21	18	15	10	15	12	8	9	6	3
1.8	28	24	20	17	12	20	17	14	9	14	11	7	9	5	3
1.9	26	22	19	16	11	19	16	13	9	13	11	7	8	5	3
2.0	25	21	18	15	11	18	15	12	8	13	10	7	8	5	3
2.25	22	19	16	14	10	16	14	11	8	11	9	6	7	4	2
2.5	21	18	15	13	9	15	13	10	7	11	9	6	7	4	2
2.75	20	17	15	12	9	14	12	10	7	10	8	5	6	4	2
3.0	19	16	14	12	8	14	12	10	6	10	8	5	6	4	2
3.5	18	16	13	11	8	13	11	9	6	9	8	5	6	4	2
4.0	18	15	13	11	8	13	11	9	6	9	7	5	6	4	2

I.3 Critical Values for the SignTest

Table I.3 Critical Values for the Sign Test Statistic S+

	Alpha								
N	***0.005***	***0.01***	***0.025***	***0.05***	***0.1***	***0.2***	***0.3***	***0.4***	***0.5***
4	4	4	4	4	3	3	3	2	2
5	5	5	5	4	4	3	3	3	2
6	6	6	5	5	5	4	4	3	3
7	7	6	6	6	5	5	4	4	3
8	7	7	7	6	6	5	5	4	4
9	8	8	7	7	6	6	5	5	4
10	9	9	8	8	7	6	6	5	5
11	10	9	9	8	8	7	6	6	5
12	10	10	9	9	8	7	7	6	6
13	11	11	10	9	9	8	7	7	6
14	12	11	11	10	9	9	8	7	7
15	12	12	11	11	10	9	9	8	7
16	13	13	12	11	11	10	9	9	8
17	14	13	12	12	11	10	10	9	8
18	14	14	13	12	12	11	10	10	9
19	15	14	14	13	12	11	11	10	9
20	16	15	14	14	13	12	11	11	10
21	16	16	15	14	13	12	12	11	10
22	17	16	16	15	14	13	12	12	11
23	18	17	16	15	15	14	13	12	11
24	18	18	17	16	15	14	13	13	12
25	19	18	17	17	16	15	14	13	12
26	19	19	18	17	16	15	14	14	13
27	20	19	19	18	17	16	15	14	13
28	21	20	19	18	17	16	15	15	14
29	21	21	20	19	18	17	16	15	14
30	22	21	20	19	19	17	16	16	15

Table I.3 Critical Values for the Sign Test Statistic S+ (continued)

	Alpha								
N	***0.005***	***0.01***	***0.025***	***0.05***	***0.1***	***0.2***	***0.3***	***0.4***	***0.5***
31	23	22	21	20	19	18	17	16	15
32	23	23	22	21	20	18	17	17	16
33	24	23	22	21	20	19	18	17	16
34	24	24	23	22	21	19	19	18	17
35	25	24	23	22	21	20	19	18	17
36	26	25	24	23	22	21	20	19	18
37	26	26	24	23	22	21	20	19	18
38	27	26	25	24	23	22	21	20	19
39	27	27	26	25	23	22	21	20	19
40	28	27	26	25	24	23	22	21	20
41	29	28	27	26	25	23	22	21	20
42	29	28	27	26	25	24	23	22	21
43	30	29	28	27	26	24	23	22	21
44	30	30	28	27	26	25	24	23	22
45	31	30	29	28	27	25	24	23	22
46	32	31	30	29	27	26	25	24	23
47	32	31	30	29	28	26	25	24	23
48	33	32	31	30	28	27	26	25	24
49	33	33	31	30	29	27	26	25	24
50	34	33	32	31	30	28	27	26	25

For N greater than 50, the table (critical) value can be calculated from:

$$\frac{N}{2} + \frac{z}{2}\sqrt{N}$$

z is the (1-α) percentile of a standard normal distribution, which can be found on page I-10 or on page 5-28 in Table 5.2.

I.4 Critical Values for the WRS Test

Table I.4 Critical Values for the WRS test

m is the number of reference area samples and n is the number of survey unit samples.

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 2	α=0.001	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43
	α=0.005	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	40	42
	α=0.01	7	9	11	13	15	17	19	21	23	25	27	28	30	32	34	36	38	39	41
	α=0.025	7	9	11	13	15	17	18	20	22	23	25	27	29	31	33	34	36	38	40
	α=0.05	7	9	11	12	14	16	17	19	21	23	24	26	27	29	31	33	34	36	38
	α=0.1	7	8	10	11	13	15	16	18	19	21	22	24	26	27	29	30	32	33	35

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 3	α=0.001	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	56	59	62	65
	α=0.005	12	15	18	21	24	27	30	32	35	38	40	43	46	48	51	54	57	59	62
	α=0.01	12	15	18	21	24	26	29	31	34	37	39	42	45	47	50	52	55	58	60
	α=0.025	12	15	18	20	22	25	27	30	32	35	37	40	42	45	47	50	52	55	57
	α=0.05	12	14	17	19	21	24	26	28	31	33	36	38	40	43	45	47	50	52	54
	α=0.1	11	13	16	18	20	22	24	27	29	31	33	35	37	40	42	44	46	48	50

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 4	α=0.001	18	22	26	30	34	38	42	46	49	53	57	60	64	68	71	75	78	82	86
	α=0.005	18	22	26	30	33	37	40	44	47	51	54	58	61	64	68	71	75	78	81
	α=0.01	18	22	26	29	32	36	39	42	46	49	52	56	59	62	66	69	72	76	79
	α=0.025	18	22	25	28	31	34	37	41	44	47	50	53	56	59	62	66	69	72	75
	α=0.05	18	21	24	27	30	33	36	39	42	45	48	51	54	57	59	62	65	68	71
	α=0.1	17	20	22	25	28	31	34	36	39	42	45	48	50	53	56	59	61	64	67

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 5	α=0.001	25	30	35	40	45	50	54	58	63	67	72	76	81	85	89	94	98	102	107
	α=0.005	25	30	35	39	43	48	52	56	60	64	68	72	77	81	85	89	93	97	101
	α=0.01	25	30	34	38	42	46	50	54	58	62	66	70	74	78	82	86	90	94	98
	α=0.025	25	29	33	37	41	44	48	52	56	60	63	67	71	75	79	82	86	90	94
	α=0.05	24	28	32	35	39	43	46	50	53	57	61	64	68	71	75	79	82	86	89
	α=0.1	23	27	30	34	37	41	44	47	51	54	57	61	64	67	71	74	77	81	84

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 6	α=0.001	33	39	45	51	57	63	67	72	77	82	88	93	98	103	108	113	118	123	128
	α=0.005	33	39	44	49	54	59	64	69	74	79	83	88	93	98	103	107	112	117	122
	α=0.01	33	39	43	48	53	58	62	67	72	77	81	86	91	95	100	104	109	114	118
	α=0.025	33	37	42	47	51	56	60	64	69	73	78	82	87	91	95	100	104	109	113
	α=0.05	32	36	41	45	49	54	58	62	66	70	75	79	83	87	91	96	100	104	108
	α=0.1	31	35	39	43	47	51	55	59	63	67	71	75	79	83	87	91	94	98	102

Table I.4 Critical Values for the WRS Test (continued)

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 7	α=0.001	42	49	56	63	69	75	81	87	92	98	104	110	116	122	128	133	139	145	151
	α=0.005	42	49	55	61	66	72	77	83	88	94	99	105	110	116	121	127	132	138	143
	α=0.01	42	48	54	59	65	70	76	81	86	92	97	102	108	113	118	123	129	134	139
	α=0.025	42	47	52	57	63	68	73	78	83	88	93	98	103	108	113	118	123	128	133
	α=0.05	41	46	51	56	61	65	70	75	80	85	90	94	99	104	109	113	118	123	128
	α=0.1	40	44	49	54	58	63	67	72	76	81	85	90	94	99	103	108	112	117	121

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 8	α=0.001	52	60	68	75	82	89	95	102	109	115	122	128	135	141	148	154	161	167	174
	α=0.005	52	60	66	73	79	85	92	98	104	110	116	122	129	135	141	147	153	159	165
	α=0.01	52	59	65	71	77	84	90	96	102	108	114	120	125	131	137	143	149	155	161
	α=0.025	51	57	63	69	75	81	86	92	98	104	109	115	121	126	132	137	143	149	154
	α=0.05	50	56	62	67	73	78	84	89	95	100	105	111	116	122	127	132	138	143	148
	α=0.1	49	54	60	65	70	75	80	85	91	96	101	106	111	116	121	126	131	136	141

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 9	α=0.001	63	72	81	88	96	104	111	118	126	133	140	147	155	162	169	176	183	190	198
	α=0.005	63	71	79	86	93	100	107	114	121	127	134	141	148	155	161	168	175	182	188
	α=0.01	63	70	77	84	91	98	105	111	118	125	131	138	144	151	157	164	170	177	184
	α=0.025	62	69	76	82	88	95	101	108	114	120	126	133	139	145	151	158	164	170	176
	α=0.05	61	67	74	80	86	92	98	104	110	116	122	128	134	140	146	152	158	164	170
	α=0.1	60	66	71	77	83	89	94	100	106	112	117	123	129	134	140	145	151	157	162

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 10	α=0.001	75	85	94	103	111	119	128	136	144	152	160	167	175	183	191	199	207	215	222
	α=0.005	75	84	92	100	108	115	123	131	138	146	153	160	168	175	183	190	197	205	212
	α=0.01	75	83	91	98	106	113	121	128	135	142	150	157	164	171	178	186	193	200	207
	α=0.025	74	81	89	96	103	110	117	124	131	138	145	151	158	165	172	179	186	192	199
	α=0.05	73	80	87	93	100	107	114	120	127	133	140	147	153	160	166	173	179	186	192
	α=0.1	71	78	84	91	97	103	110	116	122	128	135	141	147	153	160	166	172	178	184

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 11	α=0.001	88	99	109	118	127	136	145	154	163	171	180	188	197	206	214	223	231	240	248
	α=0.005	88	98	107	115	124	132	140	148	157	165	173	181	189	197	205	213	221	229	237
	α=0.01	88	97	105	113	122	130	138	146	153	161	169	177	185	193	200	208	216	224	232
	α=0.025	87	95	103	111	118	126	134	141	149	156	164	171	179	186	194	201	208	216	223
	α=0.05	86	93	101	108	115	123	130	137	144	152	159	166	173	180	187	195	202	209	216
	α=0.1	84	91	98	105	112	119	126	133	139	146	153	160	167	173	180	187	194	201	207

Table I.4 Critical Values for the WRS Test (continued)

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 12	α=0.001	102	114	125	135	145	154	164	173	183	192	202	210	220	230	238	247	256	266	275
	α=0.005	102	112	122	131	140	149	158	167	176	185	194	202	211	220	228	237	246	254	263
	α=0.01	102	111	120	129	138	147	156	164	173	181	190	198	207	215	223	232	240	249	257
	α=0.025	100	109	118	126	135	143	151	159	168	176	184	192	200	208	216	224	232	240	248
	α=0.05	99	108	116	124	132	140	147	155	165	171	179	186	194	202	209	217	225	233	240
	α=0.1	97	105	113	120	128	135	143	150	158	165	172	180	187	194	202	209	216	224	231

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 13	α=0.001	117	130	141	152	163	173	183	193	203	213	223	233	243	253	263	273	282	292	302
	α=0.005	117	128	139	148	158	168	177	187	196	206	215	225	234	243	253	262	271	280	290
	α=0.01	116	127	137	146	156	165	174	184	193	202	211	220	229	238	247	256	265	274	283
	α=0.025	115	125	134	143	152	161	170	179	187	196	205	214	222	231	239	248	257	265	274
	α=0.05	114	123	132	140	149	157	166	174	183	191	199	208	216	224	233	241	249	257	266
	α=0.1	112	120	129	137	145	153	161	169	177	185	193	201	209	217	224	232	240	248	256

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 14	α=0.001	133	147	159	171	182	193	204	215	225	236	247	257	268	278	289	299	310	320	330
	α=0.005	133	145	156	167	177	187	198	208	218	228	238	248	258	268	278	288	298	307	317
	α=0.01	132	144	154	164	175	185	194	204	214	224	234	243	253	263	272	282	291	301	311
	α=0.025	131	141	151	161	171	180	190	199	208	218	227	236	245	255	264	273	282	292	301
	α=0.05	129	139	149	158	167	176	185	194	203	212	221	230	239	248	257	265	274	283	292
	α=0.1	128	136	145	154	163	171	180	189	197	206	214	223	231	240	248	257	265	273	282

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 15	α=0.001	150	165	178	190	202	212	225	237	248	260	271	282	293	304	316	327	338	349	360
	α=0.005	150	162	174	186	197	208	219	230	240	251	262	272	283	293	304	314	325	335	346
	α=0.01	149	161	172	183	194	205	215	226	236	247	257	267	278	288	298	308	319	329	339
	α=0.025	148	159	169	180	190	200	210	220	230	240	250	260	270	280	289	299	309	319	329
	α=0.05	146	157	167	176	186	196	206	215	225	234	244	253	263	272	282	291	301	310	319
	α=0.1	144	154	163	172	182	191	200	209	218	227	236	246	255	264	273	282	291	300	309

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 16	α=0.001	168	184	197	210	223	236	248	260	272	284	296	308	320	332	343	355	367	379	390
	α=0.005	168	181	194	206	218	229	241	252	264	275	286	298	309	320	331	342	353	365	376
	α=0.01	167	180	192	203	215	226	237	248	259	270	281	292	303	314	325	336	347	357	368
	α=0.025	166	177	188	200	210	221	232	242	253	264	274	284	295	305	316	326	337	347	357
	α=0.05	164	175	185	196	206	217	227	237	247	257	267	278	288	298	308	318	328	338	348
	α=0.1	162	172	182	192	202	211	221	231	241	250	260	269	279	289	298	308	317	327	336

Table I.4 Critical Values for the WRS Test (continued)

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 17	α=0.001	187	203	218	232	245	258	271	284	297	310	322	335	347	360	372	384	397	409	422
	α=0.005	187	201	214	227	239	252	264	276	288	300	312	324	336	347	359	371	383	394	406
	α=0.01	186	199	212	224	236	248	260	272	284	295	307	318	330	341	353	364	376	387	399
	α=0.025	184	197	209	220	232	243	254	266	277	288	299	310	321	332	343	354	365	376	387
	α=0.05	183	194	205	217	228	238	249	260	271	282	292	303	313	324	335	345	356	366	377
	α=0.1	180	191	202	212	223	233	243	253	264	274	284	294	305	315	325	335	345	355	365

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 18	α=0.001	207	224	239	254	268	282	296	309	323	336	349	362	376	389	402	415	428	441	454
	α=0.005	207	222	236	249	262	275	288	301	313	326	339	351	364	376	388	401	413	425	438
	α=0.01	206	220	233	246	259	272	284	296	309	321	333	345	357	370	382	394	406	418	430
	α=0.025	204	217	230	242	254	266	278	290	302	313	325	337	348	360	372	383	395	406	418
	α=0.05	202	215	226	238	250	261	273	284	295	307	318	329	340	352	363	374	385	396	407
	α=0.1	200	211	222	233	244	255	266	277	288	299	309	320	331	342	352	363	374	384	395

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 19	α=0.001	228	246	262	277	292	307	321	335	350	364	377	391	405	419	433	446	460	473	487
	α=0.005	227	243	258	272	286	300	313	327	340	353	366	379	392	405	419	431	444	457	470
	α=0.01	226	242	256	269	283	296	309	322	335	348	361	373	386	399	411	424	437	449	462
	α=0.025	225	239	252	265	278	290	303	315	327	340	352	364	377	389	401	413	425	437	450
	α=0.05	223	236	248	261	273	285	297	309	321	333	345	356	368	380	392	403	415	427	439
	α=0.1	220	232	244	256	267	279	290	302	313	325	336	347	358	370	381	392	403	415	426

	n =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
m = 20	α=0.001	250	269	286	302	317	333	348	363	377	392	407	421	435	450	464	479	493	507	521
	α=0.005	249	266	281	296	311	325	339	353	367	381	395	409	422	436	450	463	477	490	504
	α=0.01	248	264	279	293	307	321	335	349	362	376	389	402	416	429	442	456	469	482	495
	α=0.025	247	261	275	289	302	315	329	341	354	367	380	393	406	419	431	444	457	470	482
	α=0.05	245	258	271	284	297	310	322	335	347	360	372	385	397	409	422	434	446	459	471
	α=0.1	242	254	267	279	291	303	315	327	339	351	363	375	387	399	410	422	434	446	458

Reject the null hypothesis if the test statistic (W_r) is greater than the table (critical) value. For n or m greater than 20, the table (critical) value can be calculated from:

$$m(n+m+1)/2 \; + \; z\sqrt{nm(n+m+1)/12} \tag{I.1}$$

if there are few or no ties, and from

$$m(n+m+1)/2 \; + \; z\sqrt{\frac{nm}{12}\left[(n+m+1) - \sum_{j=1}^{g} \frac{t_j(t_j^2-1)}{(n+m)(n+m-1)}\right]} \tag{I.2}$$

if there are many ties, where g is the number of groups of tied measurements and t_j is the number of tied measurements in the jth group. z is the (1-α) percentile of a standard normal distribution, which can be found in the following table:

α	**z**
0.001	3.09
0.005	2.575
0.01	2.326
0.025	1.960
0.05	1.645
0.1	1.282

Other values can be found in Table I-1.

I.5 Probability of Detecting an Elevated Area

Table I.5 Risk that an Elevated Area with Length L/G and Shape S will not be Detected and the Area (%) of the Elevated Area Relative to a Triangular Sample Grid Area of 0.866 G^2

	Shape Parameter, S																			
	0.10		0.20		0.30		0.40		0.50		0.60		0.70		0.80		0.90		1.00	
L/G	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**
0.01	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%
0.02	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%
0.03	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%
0.04	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	0.99	1%	0.99	1%
0.05	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%
0.06	1.00	<1%	1.00	<1%	1.00	<1%	0.99	<1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%
0.07	1.00	<1%	1.00	<1%	0.99	1%	0.99	<1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%	0.98	2%	0.98	2%
0.08	1.00	<1%	1.00	<1%	0.99	1%	0.99	<1%	0.99	1%	0.99	1%	0.98	2%	0.98	2%	0.98	2%	0.98	2%
0.09	1.00	<1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%	0.98	2%	0.98	2%	0.98	2%	0.97	3%	0.97	3%
0.10	1.00	<1%	0.99	1%	0.99	1%	0.99	1%	0.98	2%	0.98	2%	0.97	3%	0.97	3%	0.97	3%	0.96	4%
0.11	1.00	<1%	0.99	1%	0.99	1%	0.98	2%	0.98	2%	0.97	3%	0.97	3%	0.96	4%	0.96	4%	0.96	4%
0.12	0.99	1%	0.99	1%	0.98	2%	0.98	2%	0.97	3%	0.97	3%	0.96	4%	0.96	4%	0.95	5%	0.95	5%
0.13	0.99	1%	0.99	1%	0.98	2%	0.98	2%	0.97	3%	0.96	4%	0.96	4%	0.95	5%	0.94	6%	0.94	6%
0.14	0.99	1%	0.99	1%	0.98	2%	0.97	3%	0.96	4%	0.96	4%	0.95	5%	0.94	6%	0.94	6%	0.93	7%
0.15	0.99	1%	0.98	2%	0.98	2%	0.97	3%	0.96	4%	0.95	5%	0.94	6%	0.93	7%	0.93	7%	0.92	8%
0.16	0.99	1%	0.98	2%	0.97	3%	0.96	4%	0.95	5%	0.94	6%	0.94	7%	0.93	7%	0.92	8%	0.91	9%
0.17	0.99	1%	0.98	2%	0.97	3%	0.96	4%	0.95	5%	0.94	6%	0.93	7%	0.92	8%	0.91	9%	0.90	10%
0.18	0.99	1%	0.98	2%	0.96	4%	0.95	5%	0.94	6%	0.93	7%	0.92	8%	0.91	9%	0.89	11%	0.88	12%
0.19	0.99	1%	0.97	3%	0.96	4%	0.95	5%	0.93	7%	0.92	8%	0.91	9%	0.90	10%	0.88	12%	0.87	13%
0.20	0.99	1%	0.97	3%	0.96	4%	0.94	6%	0.93	7%	0.91	9%	0.90	10%	0.88	12%	0.87	13%	0.85	15%
0.21	0.98	2%	0.97	3%	0.95	5%	0.94	6%	0.92	8%	0.90	10%	0.89	11%	0.87	13%	0.86	14%	0.84	16%
0.22	0.98	2%	0.96	4%	0.95	5%	0.93	7%	0.91	9%	0.89	11%	0.88	12%	0.86	14%	0.84	16%	0.82	18%
0.23	0.98	2%	0.96	4%	0.94	6%	0.92	8%	0.90	10%	0.88	12%	0.87	13%	0.85	15%	0.83	17%	0.81	19%
0.24	0.98	2%	0.96	4%	0.94	6%	0.92	8%	0.90	10%	0.87	13%	0.85	15%	0.83	17%	0.81	19%	0.79	21%
0.25	0.98	2%	0.95	5%	0.93	7%	0.91	9%	0.89	11%	0.86	14%	0.84	16%	0.82	18%	0.80	20%	0.77	23%
0.26	0.98	2%	0.95	5%	0.93	7%	0.90	10%	0.88	12%	0.85	15%	0.83	17%	0.80	20%	0.78	22%	0.75	25%
0.27	0.97	3%	0.95	5%	0.92	8%	0.89	11%	0.87	13%	0.84	16%	0.81	19%	0.79	21%	0.76	24%	0.74	26%
0.28	0.97	3%	0.94	6%	0.91	9%	0.89	11%	0.86	14%	0.83	17%	0.80	20%	0.77	23%	0.74	26%	0.72	28%
0.29	0.97	3%	0.94	6%	0.91	9%	0.88	12%	0.85	15%	0.82	18%	0.79	21%	0.76	24%	0.73	27%	0.69	31%
0.30	0.97	3%	0.93	7%	0.90	10%	0.87	13%	0.84	16%	0.80	20%	0.77	23%	0.74	26%	0.71	29%	0.67	33%

Guidance for using Table I.5 can be found in Gilbert 1987 and EPA 1989a.

Table I.5 Risk that an Elevated Area with Length L/G and Shape S will not be Detected and the Area (%) of the Elevated Area Relative to a Triangular Sample Grid Area of 0.866 G^2 (continued)

	Shape Parameter, S																			
	0.10		0.20		0.30		0.40		0.50		0.60		0.70		0.80		0.90		1.00	
L/G	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**
0.31	0.97	3%	0.93	7%	0.90	10%	0.86	14%	0.83	17%	0.79	21%	0.76	24%	0.72	28%	0.69	31%	0.65	35%
0.32	0.96	4%	0.93	7%	0.89	11%	0.85	15%	0.81	19%	0.78	22%	0.74	26%	0.70	30%	0.67	33%	0.63	37%
0.33	0.96	4%	0.92	8%	0.88	12%	0.84	16%	0.80	20%	0.76	24%	0.72	28%	0.68	32%	0.64	36%	0.61	40%
0.34	0.96	4%	0.92	8%	0.87	13%	0.83	17%	0.79	21%	0.75	25%	0.71	29%	0.66	34%	0.62	38%	0.58	42%
0.35	0.96	4%	0.91	9%	0.87	13%	0.82	18%	0.78	22%	0.73	27%	0.69	31%	0.64	36%	0.60	40%	0.56	44%
0.36	0.95	5%	0.91	9%	0.86	14%	0.81	19%	0.76	24%	0.72	28%	0.67	33%	0.62	38%	0.58	42%	0.53	47%
0.37	0.95	5%	0.90	10%	0.85	15%	0.80	20%	0.75	25%	0.70	30%	0.65	35%	0.60	40%	0.55	45%	0.50	50%
0.38	0.95	5%	0.90	10%	0.84	16%	0.79	21%	0.74	26%	0.69	31%	0.63	37%	0.58	42%	0.53	47%	0.48	52%
0.39	0.94	6%	0.89	11%	0.83	17%	0.78	22%	0.72	28%	0.67	33%	0.61	39%	0.56	44%	0.50	50%	0.45	55%
0.40	0.94	6%	0.88	12%	0.83	17%	0.77	23%	0.71	29%	0.65	35%	0.59	41%	0.54	46%	0.48	52%	0.42	58%
0.41	0.94	6%	0.88	12%	0.82	18%	0.76	24%	0.70	30%	0.63	37%	0.57	43%	0.51	49%	0.45	55%	0.39	61%
0.42	0.94	6%	0.87	13%	0.81	19%	0.74	26%	0.68	32%	0.62	38%	0.55	45%	0.49	51%	0.42	58%	0.36	64%
0.43	0.93	7%	0.87	13%	0.80	20%	0.73	27%	0.66	34%	0.60	40%	0.53	47%	0.46	54%	0.40	60%	0.33	67%
0.44	0.93	7%	0.86	14%	0.79	21%	0.72	28%	0.65	35%	0.58	42%	0.51	49%	0.44	56%	0.37	63%	0.30	70%
0.45	0.93	7%	0.85	15%	0.78	22%	0.71	29%	0.63	37%	0.56	44%	0.49	51%	0.41	59%	0.34	66%	0.27	73%
0.46	0.92	8%	0.85	15%	0.77	23%	0.69	31%	0.62	38%	0.54	46%	0.46	54%	0.39	61%	0.31	69%	0.23	77%
0.47	0.92	8%	0.84	16%	0.76	24%	0.68	32%	0.60	40%	0.52	48%	0.44	56%	0.36	64%	0.28	72%	0.20	80%
0.48	0.92	8%	0.83	17%	0.75	25%	0.67	33%	0.58	42%	0.50	50%	0.41	59%	0.33	67%	0.25	75%	0.16	84%
0.49	0.91	9%	0.83	17%	0.74	26%	0.65	35%	0.56	44%	0.48	52%	0.39	61%	0.30	70%	0.22	78%	0.13	87%
0.50	0.91	9%	0.82	18%	0.73	27%	0.64	36%	0.55	45%	0.46	54%	0.37	63%	0.27	73%	0.18	82%	0.09	91%
0.51	0.91	9%	0.81	19%	0.72	28%	0.62	38%	0.53	47%	0.43	57%	0.34	66%	0.25	75%	0.15	85%	0.07	94%
0.52	0.90	10%	0.80	20%	0.71	29%	0.61	39%	0.51	49%	0.41	59%	0.32	69%	0.22	78%	0.13	88%	0.05	98%
0.53	0.90	10%	0.80	20%	0.70	31%	0.59	41%	0.49	51%	0.39	61%	0.29	71%	0.19	82%	0.10	92%	0.03	102%
0.54	0.89	11%	0.79	21%	0.68	32%	0.58	42%	0.47	53%	0.37	63%	0.27	74%	0.17	85%	0.08	95%	0.02	106%
0.55	0.89	11%	0.78	22%	0.67	33%	0.56	44%	0.46	55%	0.35	66%	0.24	77%	0.14	88%	0.06	99%	0.01	110%
0.56	0.89	11%	0.77	23%	0.66	34%	0.55	46%	0.44	57%	0.33	68%	0.22	80%	0.12	91%	0.04	102%	0.00	114%
0.57	0.88	12%	0.77	24%	0.65	35%	0.54	47%	0.42	59%	0.31	71%	0.20	83%	0.10	94%	0.02	106%	0.00	118%
0.58	0.88	12%	0.76	24%	0.64	37%	0.52	49%	0.40	61%	0.29	73%	0.18	85%	0.08	98%	0.01	110%	0.00	122%
0.59	0.87	13%	0.75	25%	0.63	38%	0.51	51%	0.39	63%	0.27	76%	0.16	88%	0.06	101%	0.00	114%	0.00	126%
0.60	0.87	13%	0.74	26%	0.62	39%	0.49	52%	0.37	65%	0.25	78%	0.14	91%	0.04	104%	0.00	118%	0.00	131%
0.61	0.87	13%	0.73	27%	0.60	40%	0.48	54%	0.35	67%	0.23	81%	0.12	94%	0.03	108%	0.00	121%	0.00	135%
0.62	0.86	14%	0.73	28%	0.59	42%	0.46	56%	0.34	70%	0.21	84%	0.10	98%	0.02	112%	0.00	126%	0.00	139%
0.63	0.86	14%	0.72	29%	0.58	43%	0.45	58%	0.32	72%	0.20	86%	0.09	101%	0.01	115%	0.00	130%	0.00	144%
0.64	0.85	15%	0.71	30%	0.57	45%	0.43	59%	0.30	74%	0.18	89%	0.07	104%	0.00	119%	0.00	134%	0.00	149%
0.65	0.85	15%	0.70	31%	0.56	46%	0.42	61%	0.29	77%	0.16	92%	0.06	107%	0.00	123%	0.00	138%	0.00	153%

Table I.5 Risk that an Elevated Area with Length L/G and Shape S will not be Detected and the Area (%) of the Elevated Area Relative to a Triangular Sample Grid Area of $0.866G^2$ (continued)

	Shape Parameter, S																			
	0.10		0.20		0.30		0.40		0.50		0.60		0.70		0.80		0.90		1.00	
L/G	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**
0.66	0.84	16%	0.69	32%	0.55	47%	0.40	63%	0.27	79%	0.15	95%	0.05	111%	0.00	126%	0.00	142%	0.00	158%
0.67	0.84	16%	0.68	33%	0.53	49%	0.39	65%	0.25	81%	0.13	98%	0.03	114%	0.00	130%	0.00	147%	0.00	163%
0.68	0.84	17%	0.68	34%	0.52	50%	0.38	67%	0.24	84%	0.12	101%	0.02	117%	0.00	134%	0.00	151%	0.00	168%
0.69	0.83	17%	0.67	35%	0.51	52%	0.36	69%	0.22	86%	0.10	104%	0.01	121%	0.00	138%	0.00	155%	0.00	173%
0.70	0.83	18%	0.66	36%	0.50	53%	0.35	71%	0.21	89%	0.09	107%	0.01	124%	0.00	142%	0.00	160%	0.00	178%
0.71	0.82	18%	0.65	37%	0.49	55%	0.33	73%	0.20	91%	0.08	110%	0.00	128%	0.00	146%	0.00	165%	0.00	183%
0.72	0.82	19%	0.64	38%	0.48	56%	0.32	75%	0.18	94%	0.07	113%	0.00	132%	0.00	150%	0.00	169%	0.00	188%
0.73	0.81	19%	0.63	39%	0.46	58%	0.31	77%	0.17	97%	0.05	116%	0.00	135%	0.00	155%	0.00	174%	0.00	193%
0.74	0.81	20%	0.62	40%	0.45	60%	0.29	79%	0.15	99%	0.04	119%	0.00	139%	0.00	159%	0.00	179%	0.00	199%
0.75	0.80	20%	0.61	41%	0.44	61%	0.28	82%	0.14	102%	0.04	122%	0.00	143%	0.00	163%	0.00	184%	0.00	204%
0.76	0.80	21%	0.61	42%	0.43	63%	0.27	84%	0.13	105%	0.03	126%	0.00	147%	0.00	168%	0.00	189%	0.00	210%
0.77	0.79	22%	0.60	43%	0.42	65%	0.25	86%	0.12	108%	0.02	129%	0.00	151%	0.00	172%	0.00	194%	0.00	215%
0.78	0.79	22%	0.59	44%	0.40	66%	0.24	88%	0.10	110%	0.01	132%	0.00	154%	0.00	177%	0.00	199%	0.00	221%
0.79	0.78	23%	0.58	45%	0.39	68%	0.23	91%	0.09	113%	0.01	136%	0.00	158%	0.00	181%	0.00	204%	0.00	226%
0.80	0.78	23%	0.57	46%	0.38	70%	0.22	93%	0.08	116%	0.00	139%	0.00	163%	0.00	186%	0.00	209%	0.00	232%
0.81	0.77	24%	0.56	48%	0.37	71%	0.20	95%	0.07	119%	0.00	143%	0.00	167%	0.00	190%	0.00	214%	0.00	238%
0.82	0.77	24%	0.55	49%	0.36	73%	0.19	98%	0.06	122%	0.00	146%	0.00	171%	0.00	195%	0.00	220%	0.00	244%
0.83	0.76	25%	0.54	50%	0.35	75%	0.18	100%	0.05	125%	0.00	150%	0.00	175%	0.00	200%	0.00	225%	0.00	250%
0.84	0.76	26%	0.53	51%	0.33	77%	0.17	102%	0.05	128%	0.00	154%	0.00	179%	0.00	205%	0.00	230%	0.00	256%
0.85	0.75	26%	0.52	52%	0.32	79%	0.16	105%	0.04	131%	0.00	157%	0.00	183%	0.00	210%	0.00	236%	0.00	262%
0.86	0.74	27%	0.51	54%	0.31	80%	0.14	107%	0.03	134%	0.00	161%	0.00	188%	0.00	215%	0.00	241%	0.00	268%
0.87	0.74	27%	0.50	55%	0.30	82%	0.13	110%	0.02	137%	0.00	165%	0.00	192%	0.00	220%	0.00	247%	0.00	275%
0.88	0.73	28%	0.50	56%	0.29	84%	0.12	112%	0.02	140%	0.00	169%	0.00	197%	0.00	225%	0.00	253%	0.00	281%
0.89	0.73	29%	0.49	57%	0.28	86%	0.11	115%	0.01	144%	0.00	172%	0.00	201%	0.00	230%	0.00	259%	0.00	287%
0.90	0.72	29%	0.48	59%	0.27	88%	0.10	118%	0.01	147%	0.00	176%	0.00	206%	0.00	235%	0.00	264%	0.00	294%
0.91	0.72	30%	0.47	60%	0.26	90%	0.10	120%	0.01	150%	0.00	180%	0.00	210%	0.00	240%	0.00	270%	0.00	300%
0.92	0.71	31%	0.46	61%	0.25	92%	0.09	123%	0.00	154%	0.00	184%	0.00	215%	0.00	246%	0.00	276%	0.00	307%
0.93	0.71	31%	0.45	63%	0.24	94%	0.08	126%	0.00	157%	0.00	188%	0.00	220%	0.00	251%	0.00	282%	0.00	314%
0.94	0.70	32%	0.44	64%	0.23	96%	0.07	128%	0.00	160%	0.00	192%	0.00	224%	0.00	256%	0.00	288%	0.00	321%
0.95	0.69	33%	0.43	65%	0.22	98%	0.07	131%	0.00	164%	0.00	196%	0.00	229%	0.00	262%	0.00	295%	0.00	327%
0.96	0.69	33%	0.42	67%	0.21	100%	0.06	134%	0.00	167%	0.00	201%	0.00	234%	0.00	267%	0.00	301%	0.00	334%
0.97	0.68	34%	0.41	68%	0.20	102%	0.05	137%	0.00	171%	0.00	205%	0.00	239%	0.00	273%	0.00	307%	0.00	341%
0.98	0.68	35%	0.40	70%	0.19	105%	0.05	139%	0.00	174%	0.00	209%	0.00	244%	0.00	279%	0.00	314%	0.00	348%
0.99	0.67	36%	0.40	71%	0.18	107%	0.04	142%	0.00	178%	0.00	213%	0.00	249%	0.00	284%	0.00	320%	0.00	356%
1.00	0.67	36%	0.39	73%	0.17	109%	0.04	145%	0.00	181%	0.00	218%	0.00	254%	0.00	290%	0.00	326%	0.00	363%

I.6 Random Numbers

Table I.6 1,000 Random Numbers Uniformly Distributed between Zero and One

0.163601	0.647423	0.555548	0.248859	0.259801	0.718368	0.305020	0.812482	0.601951	0.973160
0.934196	0.951102	0.979831	0.132364	0.157808	0.040605	0.997626	0.896462	0.360578	0.443218
0.054552	0.965257	0.999181	0.172627	0.583713	0.852958	0.116336	0.748483	0.058602	0.738495
0.972409	0.241889	0.799991	0.926726	0.585505	0.453993	0.877990	0.947022	0.910821	0.388081
0.556401	0.621126	0.293328	0.984335	0.366531	0.912588	0.733824	0.092405	0.717362	0.423421
0.625153	0.838711	0.196153	0.630553	0.867808	0.957094	0.830218	0.783518	0.141557	0.444997
0.527330	0.124034	0.351792	0.161947	0.688925	0.140346	0.553577	0.890058	0.470457	0.566196
0.826643	0.673286	0.550827	0.885295	0.690781	0.371540	0.108632	0.090765	0.618443	0.937184
0.296068	0.891272	0.392367	0.649633	0.261410	0.523221	0.769081	0.358794	0.924341	0.167665
0.848882	0.083603	0.274621	0.268003	0.272254	0.017727	0.309463	0.445986	0.244653	0.944564
0.779276	0.484461	0.101393	0.995100	0.085164	0.611426	0.030270	0.494982	0.426236	0.270225
0.095038	0.577943	0.186239	0.267852	0.786070	0.208937	0.184565	0.826397	0.256825	0.489034
0.011672	0.844846	0.443407	0.915087	0.275906	0.883009	0.243728	0.865552	0.796671	0.314429
0.215993	0.476035	0.354717	0.883172	0.840666	0.393867	0.374810	0.222167	0.114691	0.596046
0.982374	0.101973	0.683995	0.730612	0.548200	0.084302	0.145212	0.337680	0.566173	0.592776
0.860868	0.794380	0.819422	0.752871	0.158956	0.317468	0.062387	0.909843	0.779089	0.648967
0.718917	0.696798	0.463655	0.762408	0.823097	0.843209	0.368678	0.996266	0.542048	0.663842
0.800735	0.225556	0.398048	0.437067	0.642698	0.144068	0.104212	0.675095	0.318953	0.648478
0.915538	0.711742	0.232159	0.242961	0.327863	0.156608	0.260175	0.385141	0.681475	0.978186
0.975506	0.652654	0.928348	0.513444	0.744095	0.972031	0.527368	0.494287	0.602829	0.592834
0.435196	0.272807	0.452254	0.793464	0.817291	0.828245	0.407518	0.441518	0.358966	0.619741
0.692512	0.368151	0.821543	0.583707	0.802354	0.133831	0.569521	0.474516	0.437608	0.961559
0.678823	0.930602	0.657348	0.025057	0.294093	0.499623	0.006423	0.290613	0.325204	0.044439
0.642075	0.029842	0.289042	0.891009	0.813844	0.973093	0.952871	0.361623	0.709933	0.466955
0.174285	0.863244	0.133649	0.773819	0.891664	0.246417	0.272407	0.517658	0.132225	0.795514
0.951401	0.921291	0.210993	0.369411	0.196909	0.054389	0.364475	0.716718	0.096843	0.308418
0.186824	0.005407	0.310843	0.998118	0.725887	0.143171	0.293721	0.841304	0.661969	0.409622
0.105673	0.026338	0.878006	0.105936	0.612556	0.124601	0.922558	0.648985	0.896805	0.737256
0.801080	0.619461	0.933720	0.275881	0.637352	0.644996	0.713379	0.302687	0.904515	0.457172
0.101214	0.236405	0.945199	0.005975	0.893786	0.082317	0.648743	0.511871	0.298942	0.121573
0.177754	0.930066	0.390527	0.575622	0.390428	0.600575	0.460949	0.191600	0.910079	0.099444
0.846157	0.322467	0.156607	0.253388	0.739021	0.133498	0.293141	0.144834	0.626600	0.045169
0.812147	0.306383	0.201517	0.306651	0.827112	0.277716	0.660224	0.268538	0.518416	0.579216
0.691055	0.059046	0.104390	0.427038	0.148688	0.480788	0.026511	0.572705	0.745522	0.986078
0.483819	0.797573	0.174899	0.892670	0.118990	0.813221	0.857964	0.279164	0.883509	0.154562
0.165133	0.985134	0.214681	0.595309	0.741697	0.418602	0.301917	0.338913	0.680062	0.097350
0.281668	0.476899	0.839512	0.057760	0.474156	0.898409	0.482638	0.198725	0.888281	0.018872
0.554337	0.350955	0.942401	0.526759	0.509846	0.408165	0.800079	0.789263	0.564192	0.140684

Table I.6 1,000 Random Numbers Uniformly Distributed between Zero and One (continued)

0.873143	0.349662	0.238282	0.383195	0.568383	0.298471	0.490431	0.731405	0.339906	0.431645
0.401675	0.061151	0.771468	0.795760	0.365952	0.221234	0.947374	0.375686	0.828215	0.113060
0.574987	0.154831	0.808117	0.723544	0.134014	0.360957	0.166572	0.112314	0.242857	0.309290
0.745415	0.929459	0.425406	0.118845	0.386382	0.867386	0.808757	0.009573	0.229879	0.849242
0.613554	0.926550	0.857632	0.014438	0.004214	0.592513	0.280223	0.283447	0.943793	0.205750
0.880368	0.303741	0.247850	0.341580	0.867155	0.542130	0.473418	0.650251	0.326222	0.036285
0.567556	0.183534	0.696381	0.373333	0.716762	0.526636	0.306862	0.904790	0.151931	0.328792
0.280015	0.237361	0.336240	0.424191	0.192603	0.770194	0.284572	0.992475	0.308979	0.698329
0.502862	0.818555	0.238758	0.057148	0.461531	0.904929	0.521982	0.599127	0.239509	0.424858
0.738375	0.794328	0.305231	0.887161	0.021104	0.469779	0.913966	0.266514	0.647901	0.246223
0.366209	0.749763	0.634971	0.261038	0.869115	0.787951	0.678287	0.667142	0.216531	0.763214
0.739267	0.554299	0.979969	0.489597	0.545130	0.931869	0.096443	0.374089	0.140070	0.840563
0.375690	0.866922	0.256930	0.518074	0.217373	0.027043	0.801938	0.040364	0.624283	0.292810
0.894101	0.178824	0.443631	0.110614	0.556232	0.969563	0.291364	0.695764	0.306903	0.303885
0.668169	0.296926	0.324041	0.616290	0.799426	0.372555	0.070954	0.045748	0.505327	0.027722
0.470107	0.135634	0.271284	0.494071	0.485610	0.382772	0.418470	0.004082	0.298068	0.539847
0.047906	0.694949	0.309033	0.223989	0.008978	0.383695	0.479858	0.894958	0.597796	0.162072
0.917713	0.072793	0.107402	0.007328	0.176598	0.576809	0.052969	0.421803	0.737514	0.340966
0.839439	0.338565	0.254833	0.924413	0.871833	0.480599	0.172846	0.736102	0.471802	0.783451
0.488244	0.260352	0.129716	0.153558	0.305933	0.777100	0.111924	0.412930	0.601453	0.083217
0.488369	0.485094	0.322236	0.894264	0.781546	0.770237	0.707400	0.587451	0.571609	0.981580
0.311380	0.270400	0.807264	0.348433	0.172763	0.914856	0.011893	0.014317	0.820797	0.261767
0.028802	0.072165	0.944160	0.804761	0.770481	0.104256	0.112919	0.184068	0.940946	0.238087
0.466082	0.603884	0.959713	0.547834	0.487552	0.455150	0.240324	0.428921	0.648821	0.277620
0.720229	0.575779	0.939622	0.234554	0.767389	0.735335	0.941002	0.794021	0.291615	0.165732
0.861579	0.778039	0.331677	0.608231	0.646094	0.498720	0.140520	0.259197	0.782477	0.922273
0.849884	0.917789	0.816247	0.572502	0.753757	0.857324	0.988330	0.597085	0.186087	0.771997
0.989999	0.994007	0.349735	0.954437	0.741124	0.791852	0.986074	0.444554	0.177531	0.743725
0.337214	0.987184	0.344245	0.039033	0.549585	0.688526	0.225470	0.556251	0.157058	0.681447
0.706330	0.082994	0.299909	0.613361	0.031334	0.941102	0.772731	0.198070	0.460602	0.778659
0.417239	0.916556	0.707773	0.249767	0.169301	0.914420	0.732687	0.934912	0.985594	0.726957
0.653326	0.529996	0.305465	0.181747	0.153359	0.353168	0.673377	0.448970	0.546347	0.885438
0.099373	0.156385	0.067157	0.755573	0.689979	0.494021	0.996216	0.051811	0.049321	0.595525
0.860299	0.210143	0.026232	0.838499	0.108975	0.455260	0.320633	0.150619	0.445073	0.275619
0.067160	0.791992	0.363875	0.825052	0.047561	0.311194	0.447486	0.971659	0.876616	0.455018
0.944317	0.348844	0.210015	0.769274	0.253032	0.239894	0.208165	0.600014	0.945046	0.505316
0.917419	0.185575	0.743859	0.655124	0.185320	0.237660	0.271534	0.949825	0.441666	0.811135
0.365705	0.800723	0.116707	0.386073	0.837800	0.244896	0.337304	0.869528	0.845737	0.194553
0.911453	0.591254	0.920222	0.707522	0.782902	0.092884	0.426444	0.320336	0.226369	0.377845

Table I.6 1,000 Random Numbers Uniformly Distributed between Zero and One (continued)

0.027171	0.058193	0.726183	0.057705	0.935493	0.688071	0.752543	0.932781	0.048914	0.591035
0.768066	0.387888	0.655990	0.690208	0.746739	0.936409	0.685458	0.090931	0.242120	0.067899
0.052305	0.899285	0.092643	0.058916	0.826653	0.772790	0.785028	0.967761	0.588503	0.896590
0.623285	0.492051	0.644294	0.821341	0.600824	0.901289	0.774379	0.391874	0.810022	0.437879
0.624284	0.308522	0.208541	0.297156	0.576129	0.373705	0.370345	0.372748	0.965550	0.874416
0.853117	0.671602	0.018316	0.095780	0.871263	0.885420	0.919787	0.439594	0.460586	0.629443
0.967796	0.933631	0.397054	0.682343	0.505977	0.406611	0.539543	0.066152	0.885414	0.857606
0.759450	0.768853	0.115419	0.744466	0.607572	0.179839	0.413809	0.228607	0.362857	0.826932
0.514703	0.108915	0.864053	0.076280	0.352557	0.674917	0.572689	0.588574	0.596215	0.639101
0.826296	0.264540	0.255775	0.180449	0.405715	0.740170	0.423514	0.537793	0.877436	0.512284
0.354198	0.792775	0.051583	0.806962	0.385851	0.655314	0.046701	0.860466	0.848112	0.515684
0.744807	0.960789	0.123099	0.163569	0.621969	0.571558	0.482449	0.346358	0.795845	0.207558
0.642312	0.356643	0.797708	0.505570	0.418534	0.634642	0.033111	0.393330	0.105093	0.328848
0.824625	0.855876	0.770743	0.678619	0.927298	0.204828	0.831460	0.979875	0.566627	0.056160
0.755877	0.679791	0.442388	0.899944	0.563383	0.197074	0.679568	0.244433	0.786084	0.337991
0.625370	0.967123	0.321605	0.697578	0.122418	0.475395	0.068207	0.070374	0.353248	0.461960
0.124012	0.133851	0.761154	0.501578	0.204221	0.866481	0.925783	0.329001	0.327832	0.844681
0.825392	0.382001	0.847909	0.520741	0.404959	0.308849	0.418976	0.972838	0.452438	0.600528
0.999194	0.297058	0.617183	0.570478	0.875712	0.581618	0.284410	0.405575	0.362205	0.427077
0.536855	0.667083	0.636883	0.043774	0.113509	0.980045	0.237797	0.618925	0.670767	0.814902
0.361632	0.797162	0.136063	0.487575	0.682796	0.952708	0.759989	0.058556	0.292400	0.871674
0.923253	0.479871	0.022855	0.673915	0.733795	0.811955	0.417970	0.095675	0.831670	0.043950
0.845432	0.202336	0.348421	0.050704	0.171916	0.600557	0.284838	0.606715	0.758190	0.394811

I.7 Stem and Leaf Display

The construction of a **stem and leaf display** is a simple way to generate a crude histogram of the data quickly. The “stems” of such a display are the most significant digits of the data. Consider the sample data of Section 8.2.2.2:

90.7, 83.5, 86.4, 88.5, 84.4, 74.2, 84.1, 87.6, 78.2, 77.6,
86.4, 76.3, 86.5, 77.4, 90.3, 90.1, 79.1, 92.4, 75.5, 80.5.

Here the data span three decades, so one might consider using the stems 70, 80 and 90. However, three is too few stems to be informative, just as three intervals would be too few for constructing a histogram. Therefore, for this example, each decade is divided into two parts. This results in the six stems 70, 75, 80, 85, 90, 95. The leaves are the least significant digits, so 90.7 has the stem 90 and the leaf 0.7. 77.4 has the stem 75 and the leaf 7.4. Note that even though the stem is 75, the leaf is *not* 2.4. The leaf is kept as 7.4 so that the data can be read directly from the display without any calculations.

As shown in the top part of Figure I.1, simply arrange the leaves of the data into rows, one stem per row. The result is a quick histogram of the data. In order to ensure this, the same number of digits should be used for each leaf, so that each occupies the same amount of horizontal space.

If the stems are arranged in increasing order, as shown in the bottom half of Figure I.1, it is easy to pick out the minimum (74.2), the maximum (92.4), and the median (between 84.1 and 84.4).

A stem and leaf display (or histogram) with two peaks may indicate that residual radioactivity is distributed over only a portion of the survey unit. Further information on the construction and interpretation of data plots is given in EPA QA/G-9 (EPA 1996a).

Stem	Leaves
70	4.2
75	8.2, 7.6, 6.3, 7.4, 9.1, 5.5
80	3.5, 4.4, 4.1, 0.5
85	6.4, 8.5, 7.6, 6.4, 6.5
90	0.7, 0.3, 0.1, 2.4
95	

Stem	Sorted Leaves
70	4.2
75	5.5, 6.3, 7.4, 7.6, 8.2, 9.1
80	0.5, 3.5, 4.1, 4.4
85	6.4, 6.4, 6.5, 7.6, 8.5
90	0.1, 0.3, 0.7, 2.4
95	

Figure I.1 Example of a Stem and Leaf Display

I.8 Quantile Plots

A **Quantile plot** is constructed by first ranking the data from smallest to largest. Sorting the data is easy once the stem and leaf display has been constructed. Then, each data value is simply plotted against the percentage of the samples with that value or less. This percentage is computed from:

$$Percent = \frac{100\,(rank - 0.5)}{(number\ \ of\ \ data\ \ points)} \qquad \text{(I-3)}$$

The results for the example data of Section I.7 are shown in Table I.7. The Quantile plot for this example is shown in Figure I.2.

The slope of the curve in the Quantile plot is an indication of the amount of data in a given range of values. A small amount of data in a range will result in a large slope. A large amount of data in a range of values will result in a more horizonal slope. A sharp rise near the bottom or the top is an indication of asymmetry. Sudden changes in slope, or notably flat or notably steep areas may indicate peculiarities in the survey unit data needing further investigation.

Table I.7 Data for Quantile Plot

Data:	74.2	75.5	76.3	77.4	77.6	78.2	79.1	80.5	83.5	84.1
Rank:	1	2	3	4	5	6	7	8	9	10
Percent:	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5
Data:	84.4	86.4	86.4	86.5	87.6	88.5	90.1	90.3	90.7	92.4
Rank:	11	12.5	12.5	14	15	16	17	18	19	20
Percent:	52.5	60.0	60.0	67.5	72.5	77.5	82.5	87.5	92.5	97.5

A useful aid to interpreting the quantile plot is the addition of boxes containing the middle 50% and middle 75% of the data. These are shown as the dashed lines in Figure I.2. The 50% box has its upper right corner at the 75th percentile and its lower left corner at the 25th percentile. These points are also called the Quartiles. These are ~78 and ~88, respectively, as indicated by the dashed lines. They bracket the middle half of the data values. The 75% box has its upper right corner at the 87.5th percentile and its lower left corner at the 12.5th percentile. A sharp increase within the 50% box can indicate two or more modes in the data. Outside the 75% box, sharp increases can indicate outliers. The median (50th percentile) is indicated by the heavy solid line at the value ~84, and can be used as an aid to judging the symmetry of the data distribution. There are no especially unusual features in the example Quantile plot shown in Figure I.2, other than the possibility of slight asymmetry around the median.

Another Quantile plot, for the example data of Section 8.3.3, is shown in Figure I.3.

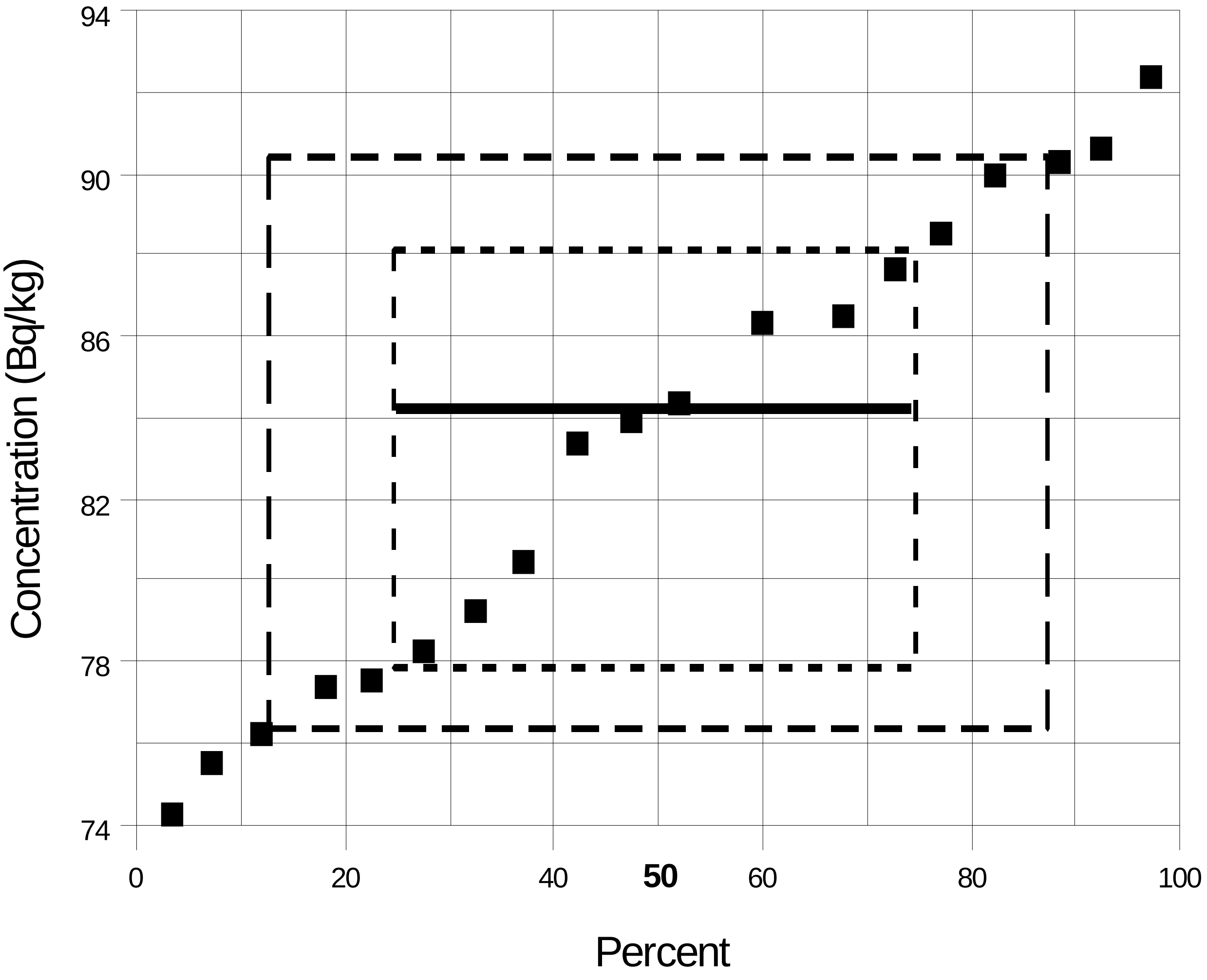

Figure I.2 Example of a Quantile Plot

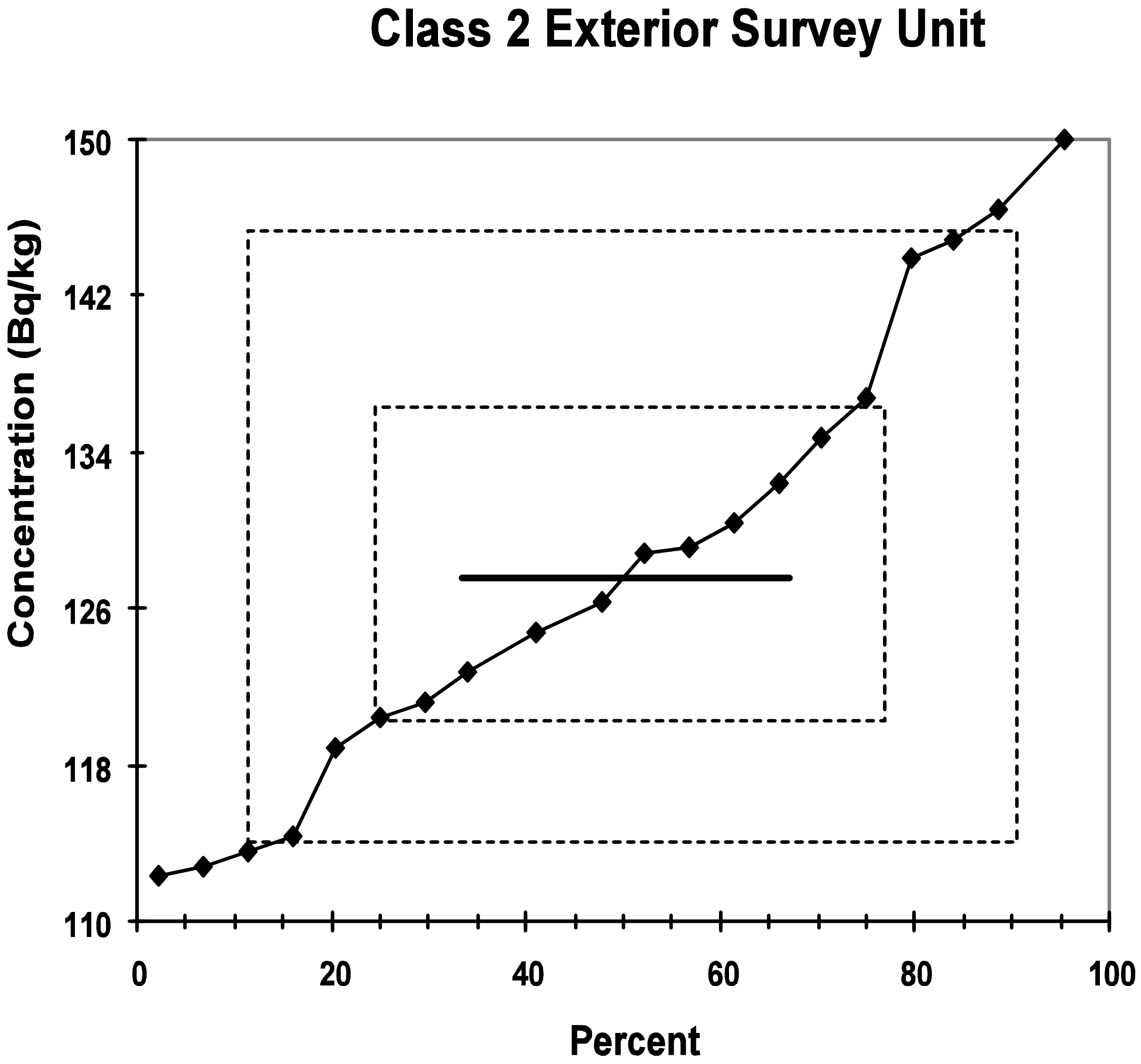

Figure I.3 Quantile Plot for Example Class 2 Exterior Survey Unit of Section 8.3.3.

A **Quantile-Quantile plot** is extremely useful for comparing two sets of data. Suppose the following 17 concentration values were obtained in a reference area corresponding to the example survey unit data of Section I.7:

92.1, 83.2, 81.7, 81.8, 88.5, 82.4, 81.5, 69.7, 82.4, 89.7,
81.4, 79.4, 82.0, 79.9, 81.1, 59.4, 75.3.

A Quantile-Quantile plot can be constructed to compare the distribution of the survey unit data, Y_j, j=1,...n, with the distribution of the reference area data X_i , i=1,... m. (If the reference area data set were the larger, the roles of X and Y would be reversed.) The data from each set are ranked separately from smallest to largest. This has already been done for the survey unit data in Table I.7. For the reference area data, we obtain the results in Table I.8.

Table I.8 Ranked Reference Area Concentrations

Data:	59.4	69.7	75.3	79.4	79.9	81.1	81.4	81.5	81.7	81.8
Rank:	1	2	3	4	5	6	7	8	9	10
Data:	82.0	82.4	82.4	83.2	88.5	89.7	92.1			
Rank:	11	12.5	12.5	14	15	16	17			

The median for the reference area data is 81.7, the sample mean is 80.7, and the sample standard deviation is 7.5.

For the larger data set, the data must be interpolated to match the number of points in the smaller data set. This is done by computing

$$\nu_1 = 0.5(n/m) + 0.5 \quad and \quad \nu_{i+1} = \nu_i + (n/m) \quad for\ i = 1,...m-1, \tag{I-4}$$

where m is the number of points in the smaller data set and n is the number of points in the larger data set. For each of the ranks, i, in the smaller data set, a corresponding value in the larger data set is found by first decomposing ν_i into its integer part, j, and its fractional part, g.

Then the interpolated values are computed from the relationship:

$$Z_i = (1\text{-}g)\, Y_j + g\, Y_{j+1}\,. \qquad \text{(I-5)}$$

The results of these calculations are shown in Table I.9.

Table I.9 Interpolated Ranks for Survey Unit Concentrations

Rank	1	2	3	4	5	6	7	8	9	10
v_i	1.09	2.26	3.44	4.62	5.79	6.97	8.15	9.33	10.50	11.68
Z_i	74.3	75.7	76.8	77.5	78.1	79.1	80.9	83.7	84.3	85.8
X_i	59.4	69.7	75.3	79.4	79.7	81.1	81.4	81.5	81.7	81.8
Rank	11	12.5	12.5	14	15	16	17			
v_i	12.85	14.03	15.21	16.38	17.56	18.74	19.91			
Z_i	86.4	86.5	87.8	89.1	90.2	90.6	92.3			
X_i	82.0	82.4	82.4	83.2	88.5	89.7	92.1			

Finally, Z_i is plotted against X_i to obtain the Quantile-Quantile plot. This example is shown in Figure I.4.

The Quantile-Quantile Plot is valuable because it provides a direct visual comparison of the two data sets. If the two data distributions differ only in location (*e.g.* mean) or scale (*e.g.* standard deviation), the points will lie on a straight line. If the two data distributions being compared are identical, all of the plotted points will lie on the line Y=X. Any deviations from this would point to possible differences in these distributions. The middle data point plots the median of Y against the median of X. That this point lies above the line Y=X, in the example of Figure 8.4, shows that the median of Y is larger than the median of X. Indeed, the cluster of points above the line $Y = X$ in the region of the plot where the data points are dense, is an indication that the central portion of the survey unit distribution is shifted toward higher values than the reference area distribution. This could imply that there is residual radioactivity in the survey unit. This should be tested using the nonparametric statistical tests described in Chapter 8.

Another Quantile-Quantile plot, for the Class 1 Interior Survey Unit example data, is shown in Figure A.8.

Further information on the interpretation of Quantile and Quantile-Quantile plots are given in EPA QA/G-9 (EPA 1996a).

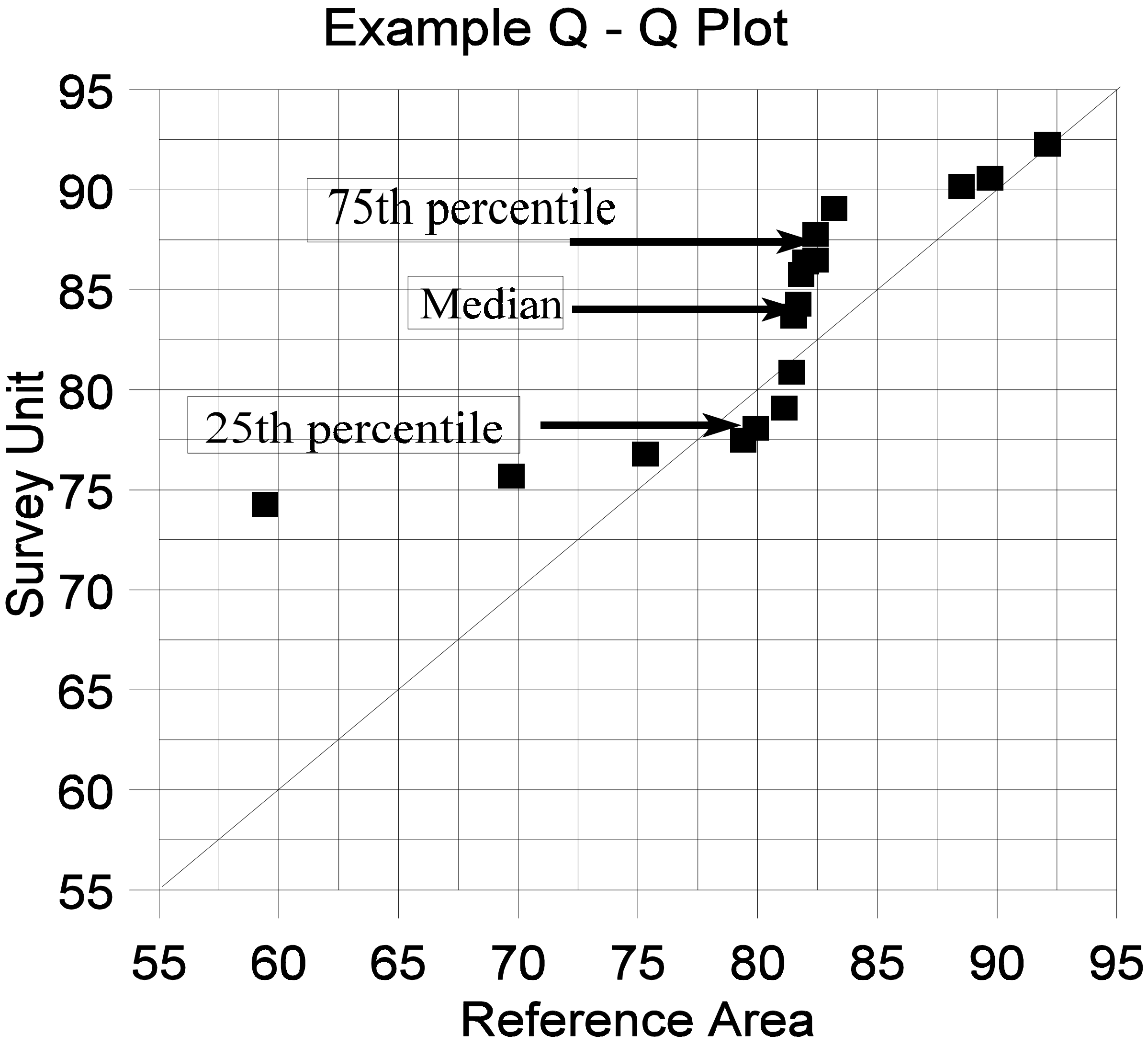

Figure I.4 Example Quantile-Quantile Plot

I.9 Power Calculations for the Statistical Tests

I.9.1 Power of the Sign Test

The power of the Sign test for detecting residual radioactivity at the concentration level LBGR = DGCL - Δ, may be found using equation I-6.

$$1 - \beta = 1 - \sum_{i=0}^{k} \binom{N}{i} [q^*]^i \cdot [1 - q^*]^{N-i} \approx 1 - \Phi\left(\frac{k - Nq^*}{\sqrt{Nq^*(1-q^*)}} \right) \qquad \text{(I-6)}$$

with

$$q^* = \Phi(\Delta/\sigma) \qquad \text{(I-7)}$$

The function Φ(z) is the standard normal cumulative distribution function tabulated in Table I.1. Note that if Δ/σ is large, q^* approaches one, and the power also approaches one. This calculation can be performed for other values, Δ***,** in order to construct a power curve for the test. These calculations can also be performed using the standard deviation of the actual measurement data, *s*, in order to construct a retrospective power curve for the test. This is an important step when the null hypothesis is not rejected, since it demonstrates whether the DQOs have been met.

The retrospective power curve for the Sign test can be constructed using Equations I-6 and I-7, together with the actual number of concentration measurements obtained, N. The power as a function of Δ/σ is calculated. The values of Δ/σ are converted to concentration using:

Concentration = DCGL_W - (Δ/σ)(observed standard deviation).

The results for the Class 3 Exterior Survey Unit example of Section 8.3.4 are plotted in Figure I.5. This figure shows the probability that the survey unit would have passed the release criterion using the Sign test versus concentration of residual radioactivity. This curve shows that the data quality objectives were met, despite the fact that the actual standard deviation was larger than that used in designing the survey. This is primarily due to the additional 20% that was added to the sample size, and also that sample sizes were always rounded up. The curve shows that a survey unit with less than 135 Bq/kg would almost always pass, and that a survey unit with more than 145 Bq/kg would almost always fail.

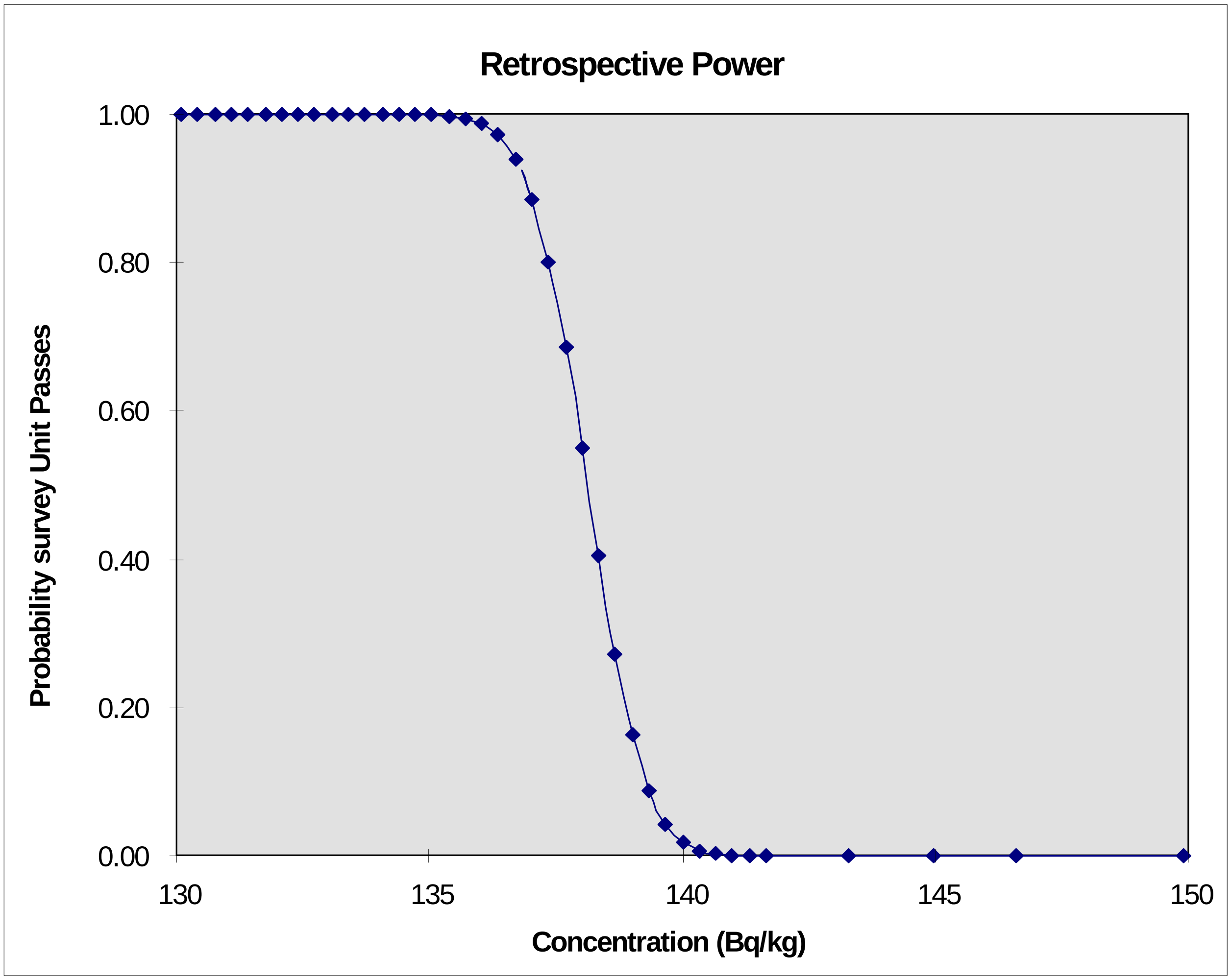

Figure I.5 Retrospective Power Curve for Class 3 Exterior Survey Unit

I.9.2 Power of the Wilcoxon Rank Sum Test

The power of the WRS test is computed from

$$Power = 1 - \Phi[\frac{W_c - 0.5 - 0.5m(m+1) - E(W_{MW})}{\sqrt{Var(W_{MW})}}] \tag{I-8}$$

where W_c is the critical value found in Table I.4 for the appropriate vales of α, *n* and *m*. Values of Φ(*z*), the standard normal cumulative distribution function, are given in Table I.1.

$W_{MW} = W_r - 0.5m(m+1)$ is the Mann-Whitney form of the WRS test statistic. Its mean is

$$E(W_{MW}) = mnP_r \tag{I-9}$$

and its variance is

$$\mathrm{Var}(W_{MW}) = mnP_r(1-P_r) + mn(n+m-2)(p_2 - P_r^2) \tag{I-10}$$

Values of P_r and p_2 as a function of Δ/σ are given in Table I.10.

The power calculated in Equation I-8 is an approximation, but the results are generally accurate enough to be used to determine if the sample design achieves the DQOs.

The retrospective power curve for the WRS test can be constructed using Equations I-8, I-9, and I-10, together with the actual number of concentration measurements obtained, N. The power as a function of Δ/σ is calculated. The values of Δ/σ are converted to dpm/100 cm^2 using:

dpm/100 cm^2 = DCGL - (Δ/σ)(observed standard deviation).

The results for this example are plotted in Figure I.6, showing the probability that the survey unit would have passed the release criterion using the WRS test versus dpm of residual radioactivity. This curve shows that the data quality objectives were easily achieved. The curve shows that a survey unit with less than 4,500 dpm/100 cm^2 above background would almost always pass, and that one with more than 5,100 dpm/100 cm^2 above background would almost always fail.

Table I.10 Values of P_r and p_2 for Computing the Mean and Variance of W_{MW}

Δ/σ	P_r	p_2	Δ/σ	P_r	p_2
-6.0	1.11E-05	1.16E-07	0.7	0.689691	0.544073
-5.0	0.000204	6.14E-06	0.8	0.714196	0.574469
-4.0	0.002339	0.000174	0.9	0.737741	0.604402
-3.5	0.006664	0.000738	1.0	0.760250	0.633702
-3.0	0.016947	0.002690	1.1	0.781662	0.662216
-2.5	0.038550	0.008465	1.2	0.801928	0.689800
-2.0	0.078650	0.023066	1.3	0.821015	0.716331
-1.9	0.089555	0.027714	1.4	0.838901	0.741698
-1.8	0.101546	0.033114	1.5	0.855578	0.765812
-1.7	0.114666	0.039348	1.6	0.871050	0.788602
-1.6	0.128950	0.046501	1.7	0.885334	0.810016
-1.5	0.144422	0.054656	1.8	0.898454	0.830022
-1.4	0.161099	0.063897	1.9	0.910445	0.848605
-1.3	0.178985	0.074301	2.0	0.921350	0.865767
-1.2	0.198072	0.085944	2.1	0.931218	0.881527
-1.1	0.218338	0.098892	2.2	0.940103	0.895917
-1.0	0.239750	0.113202	2.3	0.948062	0.908982
-0.9	0.262259	0.128920	2.4	0.955157	0.920777
-0.8	0.285804	0.146077	2.5	0.961450	0.931365
-0.7	0.310309	0.164691	2.6	0.967004	0.940817
-0.6	0.335687	0.184760	2.7	0.971881	0.949208
-0.5	0.361837	0.206266	2.8	0.976143	0.956616
-0.4	0.388649	0.229172	2.9	0.979848	0.963118
-0.3	0.416002	0.253419	3.0	0.983053	0.968795
-0.2	0.443769	0.278930	3.1	0.985811	0.973725
-0.1	0.471814	0.305606	3.2	0.988174	0.977981
0.0	0.500000	0.333333	3.3	0.990188	0.981636
0.1	0.528186	0.361978	3.4	0.991895	0.984758
0.2	0.556231	0.391392	3.5	0.993336	0.987410
0.3	0.583998	0.421415	4.0	0.997661	0.995497
0.4	0.611351	0.451875	5.0	0.999796	0.999599
0.5	0.638163	0.482593	6.0	0.999989	0.999978
0.6	0.664313	0.513387			

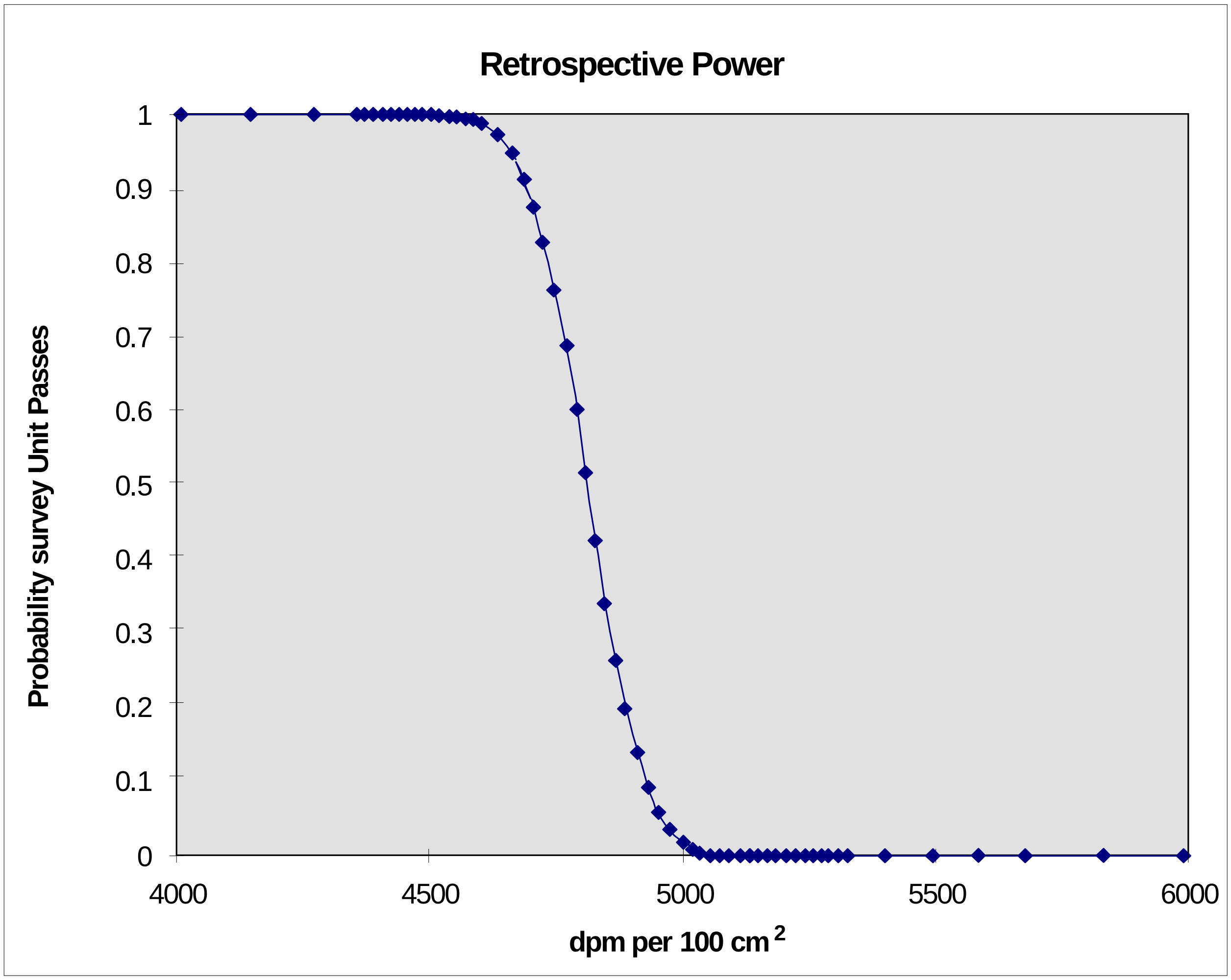

Figure I.6 Retrospective Power Curve for Class 2 Interior Drywall Survey Unit

I.10 Spreadsheet Formulas for the Wilcoxon Rank Sum Test

The analysis for the WRS test is very well suited for calculation on a spreadsheet. This is how the analysis discussed above was done. This particular example was constructed using Excel 5.0™. The formula sheet corresponding to Table 8.6 is given in Table I.11. The function in Column D of Table I.11 calculates the ranks of the data. The RANK function in Excel™ does not return tied ranks in the way needed for the WRS. The COUNTIF function is used to correct for this. Column E simply picks out the reference area ranks from Column D.

Table I.11 Spreadsheet Formulas Used in Table 8.6

	A	**B**	**C**	**D**	**E**
1	**Data**	**Area**	**Adjusted Data**	**Ranks**	**Reference Area Ranks**
2	49	R	=IF(B2="R",A2+160,A2)	=RANK(C2,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C2) - 1) / 2	=IF(B2="R",D2,0)
3	35	R	=IF(B3="R",A3+160,A3)	=RANK(C3,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C3) - 1) / 2	=IF(B3="R",D3,0)
4	45	R	=IF(B4="R",A4+160,A4)	=RANK(C4,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C4) - 1) / 2	=IF(B4="R",D4,0)
5	45	R	=IF(B5="R",A5+160,A5)	=RANK(C5,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C5) - 1) / 2	=IF(B5="R",D5,0)
6	41	R	=IF(B6="R",A6+160,A6)	=RANK(C6,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C6) - 1) / 2	=IF(B6="R",D6,0)
7	44	R	=IF(B7="R",A7+160,A7)	=RANK(C7,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C7) - 1) / 2	=IF(B7="R",D7,0)
8	48	R	=IF(B8="R",A8+160,A8)	=RANK(C8,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C8) - 1) / 2	=IF(B8="R",D8,0)
9	37	R	=IF(B9="R",A9+160,A9)	=RANK(C9,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C9) - 1) / 2	=IF(B9="R",D9,0)
10	46	R	=IF(B10="R",A10+160,A10)	=RANK(C10,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C10) - 1) / 2	=IF(B10="R",D10,0)
11	42	R	=IF(B11="R",A11+160,A11)	=RANK(C11,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C11) - 1) / 2	=IF(B11="R",D11,0)
12	47	R	=IF(B12="R",A12+160,A12)	=RANK(C12,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C12) - 1) / 2	=IF(B12="R",D12,0)
13	104	S	=IF(B13="R",A13+160,A13)	=RANK(C13,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C13) - 1) / 2	=IF(B13="R",D13,0)
14	94	S	=IF(B14="R",A14+160,A14)	=RANK(C14,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C14) - 1) / 2	=IF(B14="R",D14,0)
15	98	S	=IF(B15="R",A15+160,A15)	=RANK(C15,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C15) - 1) / 2	=IF(B15="R",D15,0)
16	99	S	=IF(B16="R",A16+160,A16)	=RANK(C16,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C16) - 1) / 2	=IF(B16="R",D16,0)
17	90	S	=IF(B17="R",A17+160,A17)	=RANK(C17,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C17) - 1) / 2	=IF(B17="R",D17,0)
18	104	S	=IF(B18="R",A18+160,A18)	=RANK(C18,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C18) - 1) / 2	=IF(B18="R",D18,0)
19	95	S	=IF(B19="R",A19+160,A19)	=RANK(C19,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C19) - 1) / 2	=IF(B19="R",D19,0)
20	105	S	=IF(B20="R",A20+160,A20)	=RANK(C20,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C20) - 1) / 2	=IF(B20="R",D20,0)
21	93	S	=IF(B21="R",A21+160,A21)	=RANK(C21,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C21) - 1) / 2	=IF(B21="R",D21,0)
22	101	S	=IF(B22="R",A22+160,A22)	=RANK(C22,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C22) - 1) / 2	=IF(B22="R",D22,0)
23	92	S	=IF(B23="R",A23+160,A23)	=RANK(C23,$C$2:$C$23,1)+(COUNTIF($C$2:$C$23,C23) - 1) / 2	=IF(B23="R",D23,0)
24			Sum=	=SUM(D2:D23)	=SUM(E2:E23)

I.11 Multiple Radionuclides

There are two cases to be considered when dealing with multiple radionuclides, namely 1) the radionuclide concentrations have a fairly constant ratio throughout the survey unit, or 2) the concentrations of the different radionuclides appear to be unrelated in the survey unit. In statistical terms, we are concerned about whether the concentrations of the different radionuclides are correlated or not. A simple way to judge this would be to make a scatter plot of the concentrations against each other, and see if the points appear to have an underlying linear pattern. The correlation coefficient can also be computed to see if it lies nearer to zero than to one. One could also perform a curve fit and test the significance of the result. Ultimately, however, sound judgement must be used in interpreting the results of such calculations. If there is no physical reason for the concentrations to be related, they probably are not. Conversely, if there is sound evidence that the radionuclide concentrations should be related because of how they were treated, processed or released, this information should be used.

I.11.1 Using the Unity Rule

In either of the two above cases, the unity rule described in Section 4.3.3 is applied. The difference is in how it is applied. Suppose there are *n* radionuclides. If the concentration of radionuclide *i* is denoted by C_i, and its DCGL_W is denoted by D_i, then the unity rule for the *n* radionuclides states that:

$$\mathrm{C}_1 / D_1 + C_2 / D_2 + C_3 / D_3 + \cdots + C_n / D_n \leq 1 \qquad \text{(I-11)}$$

This will ensure that the total dose or risk due to the sum of all the radionuclides does not exceed the release criterion. Note that if D_{min} is the smallest of the DCGLs, then

$$(C_1 + C_2 + C_3 + \cdots + C_n)/D_{min} \leq C_1 / D_1 + C_2 / D_2 + C_3 / D_3 + \cdots + C_n / D_n \qquad \text{(I-12)}$$

so that the smallest DCGL may be applied to the total activity concentration, rather than using the unity rule. While this option may be considered, in many cases it will be too conservative to be useful.

I.11.2 Radionuclide Concentrations with Fixed Ratios

If there is an established ratio among the concentrations of the *n* radionuclides in a survey unit, then the concentration of every radionuclide can be expressed in terms of any one of them, e.g., radionuclide #1. The measured radionuclide is often called a *surrogate* radionuclide for the others.

If $C_2 = R_2\, C_1,\ C_3 = R_3\, C_1, \ldots,\ C_i = R_i\, C_1,\ \ldots,\ C_n = R_n\, C_1$
then

$$\begin{aligned} & C_1 / D_1 + C_2 / D_2 + C_3 / D_3 + \cdots + C_n / D_n \\ &= C_1 / D_1 + R_2\, C_1 / D_2 + R_3\, C_1 / D_3 + \cdots + R_n\, C_1 / D_n \\ &= C_1\, [1/ D_1 + R_2 / D_2 + R_3 / D_3 + \cdots + R_n / D_n] \\ &= C_1 / D_{total} \end{aligned} \qquad \text{(I-13)}$$

where

$$D_{total} = 1/\, [1/ D_1 + R_2 / D_2 + R_3 / D_3 + \cdots + R_n / D_n\,] \qquad \text{(I-14)}$$

Thus, D_{total} is the $\mathrm{DCGL_W}$ for the surrogate radionuclide when the concentration of that radionuclide represents all radionuclides that are present in the survey unit. Clearly, this scheme is applicable only when radionuclide specific measurements of the surrogate radionuclide are made. It is unlikely to apply in situations where the surrogate radionuclide appears in background, since background variations would tend to obscure the relationships between it and the other radionuclides.

Thus, in the case where there are constant ratios among radionuclide concentrations, the statistical tests are applied as if only the surrogate radionuclide were contributing to the residual radioactivity, with the $\mathrm{DCGL_W}$ for that radionuclide replaced by D_{total}. For example, in planning the final status survey, only the expected standard deviation of the concentration measurements for the surrogate radionuclide is needed to calculate the sample size.

For the elevated measurement comparison, the $\mathrm{DCGL_{EMC}}$ for the surrogate radionuclide is replaced by

$$E_{total} = 1/\, [1/ E_1 + R_2 / E_2 + R_3 / E_3 + \cdots + R_n / E_n\,] \qquad \text{(I-15)}$$

where E_i is the $\mathrm{DCGL_{EMC}}$ for radionuclide i.

I.11.3 Unrelated Radionuclide Concentrations

If the concentrations of the different radionuclides appear to be unrelated in the survey unit, there is little alternative but to measure the concentration of each radionuclide and use the unity rule. The exception would be in applying the most restrictive $\mathrm{DCGL_W}$ to all of the radionuclides, as mentioned later in this section.

Since the release criterion is

$$\mathrm{C}_1 / D_1 + C_2 / D_2 + C_3 / D_3 + \cdots + C_n / D_n \le 1 \qquad \text{(I-16)}$$

the quantity to be measured is the *weighted sum*, $T = C_1 / D_1 + C_2 / D_2 + C_3 / D_3 + \cdots + C_n / D_n$. The $\mathrm{DCGL_W}$ for T is one. In planning the final status survey, the measurement standard deviation of the weighted sum, T, is estimated by

$$\sigma^2(T) = [\sigma(C_1)/ D_1]^2 + [\sigma(C_2)/ D_2]^2 + [\sigma(C_3)/ D_3]^2 + \cdots + [\sigma(C_n)/ D_n]^2 \qquad \text{(I-17)}$$

since the measured concentrations of the various radionuclides are assumed to be uncorrelated.

For the elevated measurement comparison, the inequality

$$C_1 / E_1 + C_2 / E_2 + C_3 / E_3 + \cdots + C_n / E_n \leq 1 \qquad \text{(I-18)}$$

is used, where E_i is the $\mathrm{DCGL_{EMC}}$ for radionuclide i. For scanning, the most restrictive $\mathrm{DCGL_{EMC}}$ should generally be used.

When some of the radionuclides also appear in background, the quantity $T = C_1 / D_1 + C_2 / D_2 + C_3 / D_3 + \cdots + C_n / D_n$ must also be measured in an appropriate reference area. If radionuclide i does not appear in background, set $C_i = 0$ in the calculation of T for the reference area.

Note that if there is a fixed ratio between the concentrations of some radionuclides, but not others, a combination of the method of this section with that of the previous section may be used. The appropriate value of D_{total} with the concentration of the measured surrogate radionuclide should replace the corresponding terms in equation I-17.

I.11.4 Example Application of WRS Test to multiple radionuclides

This section contains an example application of the nonparametric statistical methods in this report to sites that have residual radioactivity from more than one radionuclide. Consider a site with both ^{60}Co and ^{137}Cs contamination. ^{137}Cs appears in background from global atmospheric weapons tests at a typical concentration of about 1 pCi/g. Assume that the $\mathrm{DCGL_W}$ for ^{60}Co is 2 pCi/g and for ^{137}Cs is 1.4 pCi/g. In disturbed areas, the background concentration of ^{137}Cs can vary considerably. An estimated spatial standard deviation of 0.5 pCi/g for ^{137}Cs will be assumed. During remediation, it was found that the concentrations of the two radionuclides were not well correlated in the survey unit. ^{60}Co concentrations were more variable than the ^{137}Cs concentrations, and 0.7 pCi/g is estimated for its standard deviation. Measurement errors for both ^{60}Co and ^{137}Cs using gamma spectrometry will be small compared to this. For the comparison to the release criteria, the weighted sum of the concentrations of these radionuclides is computed from:

$$\begin{aligned}\text{Weighted sum} &= (^{60}\text{Co concentration})/(^{60}\text{Co DCGL}_\text{W}) + (^{137}\text{Cs Concentration})/(^{137}\text{Cs DCGL}_\text{W}) \\ &= (^{60}\text{Co concentration})/(2) + (^{137}\text{Cs Concentration})/(1.4)\end{aligned}$$

The variance of the weighted sum, assuming that the ^{60}Co and ^{137}Cs concentrations are spatially unrelated is

$$\sigma^2 = [(^{60}\text{Co Standard deviation})/(^{60}\text{Co DCGL}_W)]^2 + [(^{137}\text{Cs Standard Deviation})/(^{137}\text{Cs DCGL}_W)]^2$$
$$= [(0.7)/(2)]^2 + [(0.5)/(1.4)]^2 = 0.25.$$

Thus $\sigma = 0.5$. The DCGL_W for the weighted sum is one. The null hypothesis is that the survey unit exceeds the release criterion. During the DQO process, the LBGR was set at 0.5 for the weighted sum, so that $\Delta = \text{DCGL}_W - \text{LBGR} = 1.0 - 0.5 = 0.5$, and $\Delta/\sigma = 0.5/0.5 = 1.0$. The acceptable error rates chosen were $\alpha = \beta = 0.05$. To achieve this, 32 samples each are required in the survey unit and the reference area.

The weighted sums are computed for each measurement location in both the reference area and the survey unit. The WRS test is then performed on the weighted sum. The calculations for this example are shown in Table I.12. The DCGL_W (i.e., 1.0) is added to the weighted sum for each location in the reference area. The ranks of the combined survey unit and adjusted reference area weighted sums are then computed. The sum of the ranks of the adjusted reference area weighted sums is then compared to the critical value for $n = m = 32$, $\alpha = 0.05$, which is 1162 (see formula following Table I.4). In Table I.12, the sum of the ranks of the adjusted reference area weighted sums is 1281. This exceeds the critical value, so the null hypothesis is rejected. The survey unit meets the release criterion. The difference between the mean of the weighted sums in the survey unit and the reference area is 1.86 - 1.16 = 0.7. Thus, the estimated dose or risk due to residual radioactivity in the survey unit is 70% of the release criterion.

Table I.12 Example WRS Test for Two Radionuclides

	Reference Area		**Survey Unit**		**Weighted Sum**			**Ranks**	
	^{137}*Cs*	^{60}*Co*	^{137}*Cs*	^{60}*Co*	*Ref*	*Survey*	*Adj Ref*	*Survey*	*Adj Ref*
1	2.00	0	1.12	0.06	1.43	0.83	2.43	1	56
2	1.23	0	1.66	1.99	0.88	2.18	1.88	43	21
3	0.99	0	3.02	0.56	0.71	2.44	1.71	57	14
4	1.98	0	2.47	0.26	1.41	1.89	2.41	23	55
5	1.78	0	2.08	0.21	1.27	1.59	2.27	9	50
6	1.93	0	2.96	0.00	1.38	2.11	2.38	37	54
7	1.73	0	2.05	0.20	1.23	1.56	2.23	7	46
8	1.83	0	2.41	0.00	1.30	1.72	2.30	16	52
9	1.27	0	1.74	0.00	0.91	1.24	1.91	2	24
10	0.74	0	2.65	0.16	0.53	1.97	1.53	27	6
11	1.17	0	1.92	0.63	0.83	1.68	1.83	13	18
12	1.51	0	1.91	0.69	1.08	1.71	2.08	15	32
13	2.25	0	3.06	0.13	1.61	2.25	2.61	47	63
14	1.36	0	2.18	0.98	0.97	2.05	1.97	30	28
15	2.05	0	2.08	1.26	1.46	2.12	2.46	39	58
16	1.61	0	2.30	1.16	1.15	2.22	2.15	45	41
17	1.29	0	2.20	0.00	0.92	1.57	1.92	8	25
18	1.55	0	3.11	0.50	1.11	2.47	2.11	59	35
19	1.82	0	2.31	0.00	1.30	1.65	2.30	11	51
20	1.17	0	2.82	0.41	0.84	2.22	1.84	44	19
21	1.76	0	1.81	1.18	1.26	1.88	2.26	22	48
22	2.21	0	2.71	0.17	1.58	2.02	2.58	29	62
23	2.35	0	1.89	0.00	1.68	1.35	2.68	3	64
24	1.51	0	2.12	0.34	1.08	1.68	2.08	12	33
25	0.66	0	2.59	0.14	0.47	1.92	1.47	26	5
26	1.56	0	1.75	0.71	1.12	1.60	2.12	10	38
27	1.93	0	2.35	0.85	1.38	2.10	2.38	34	53
28	2.15	0	2.28	0.87	1.54	2.06	2.54	31	61
29	2.07	0	2.56	0.56	1.48	2.11	2.48	36	60
30	1.77	0	2.50	0.00	1.27	1.78	2.27	17	49
31	1.19	0	1.79	0.30	0.85	1.43	1.85	4	20
32	1.57	0	2.55	0.70	1.12	2.17	2.12	42	40
Avg	1.62	0	2.28	0.47	1.16	1.86	2.16	sum = 799	sum = 1281
Std Dev	0.43	0	0.46	0.48	0.31	0.36	0.31		